MHT CET 2023 : 12th May Morning Shift

Mathematics

Question 1

If $\overline{\mathrm{a}}, \overline{\mathrm{b}}, \overline{\mathrm{c}}$ are three vectors, $|\overline{\mathrm{a}}|=2, |\overline{\mathrm{b}}|=4, |\overline{\mathrm{c}}|=1, |\bar{b} \times \bar{c}|=\sqrt{15}$ and $\bar{b}=2 \bar{c}+\lambda \bar{a}$, then the value of λ is

Options:

A. 2

B. $2 \sqrt{2}$

C. 1

D. 4

Answer: A

Solution:

If angle between $\bar{b}$ and $\bar{c}$ is α and $|\bar{b} \times \bar{c}|=\sqrt{15}$

$$\begin{aligned}
& \Rightarrow|\bar{b}||\bar{c}| \sin \alpha=\sqrt{15} \\
& \Rightarrow \sin \alpha=\frac{\sqrt{15}}{4} \\
& \Rightarrow \cos \alpha=\frac{1}{4}
\end{aligned}$$

Now, $\overline{\mathrm{b}}-2 \overline{\mathrm{c}}=\lambda \overline{\mathrm{a}}$

$$\begin{aligned}
& \Rightarrow|\bar{b}-2 \bar{c}|^2=\lambda^2|\bar{a}|^2 \\
& \Rightarrow|\bar{b}|^2+4|\bar{c}|^2-4 \bar{b} \cdot \bar{c}=\lambda^2|\bar{a}|^2 \\
& \Rightarrow 16+4-4\{|\bar{b}||\bar{c}| \cos \alpha\}=4 \lambda^2 \\
& \Rightarrow 16+4-4 \times 4 \times 1 \times \frac{1}{4}=4 \lambda^2 \\
& \Rightarrow 4 \lambda^2=16 \\
& \Rightarrow \lambda= \pm 2
\end{aligned}$$

Question 2

The centroid of tetrahedron with vertices at $\mathrm{A}(-1,2,3), \mathrm{B}(3,-2,1), \mathrm{C}(2,1,3)$ and $\mathrm{D}(-1,-2,4)$ is

Options:

A. $\left(\frac{3}{4}, \frac{-1}{4}, \frac{11}{4}\right)$

B. $\left(\frac{5}{4}, \frac{-3}{4}, \frac{7}{4}\right)$

C. $\left(\frac{-3}{4}, \frac{-1}{4}, \frac{11}{4}\right)$

D. $\left(\frac{-5}{4}, \frac{-3}{4}, \frac{-7}{4}\right)$

Answer: A

Solution:

Centroid of tetrahedron

$$\equiv\left(\frac{-1+3+2-1}{4}, \frac{2-2+1-2}{4}, \frac{3+1+3+4}{4}\right)$$
$$\equiv\left(\frac{3}{4}, \frac{-1}{4}, \frac{11}{4}\right)$$

Question 3

Two adjacent sides of a parallelogram ABCD are given by $\overline{AB}=2\hat{i}+10\hat{j}+11\hat{k}$ and $\overline{\mathrm{AD}}=-\hat{\mathrm{i}}+2\hat{\mathrm{j}}+2\hat{\mathrm{k}}$. The side AD is rotated by an acute angle α in the plane of parallelogram so that AD becomes AD'. If AD' makes a right angle with side AB, then the cosine of the angle α is given by

Options:

A. $\frac{8}{9}$

B. $\frac{\sqrt{17}}{9}$

C. $\frac{1}{9}$

D. $\frac{4\sqrt{5}}{9}$

Answer: B

Solution:

Let θ be the angle between $\overline{\mathrm{AB}}$ and $\overline{\mathrm{AD}}$

$$\begin{aligned} \therefore \quad \cos\theta &= \frac{\overline{\mathrm{AB}} \cdot \overline{\mathrm{AD}}}{|\overline{\mathrm{AB}}||\overline{\mathrm{AD}}|} \\ &= \frac{(2\hat{\mathrm{i}} + 10\hat{\mathrm{j}} + 11\hat{\mathrm{k}}) \cdot (-\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 2\hat{\mathrm{k}})}{\sqrt{4+100+121}\sqrt{1+4+4}} \\ &= \frac{-2+20+22}{\sqrt{225}\sqrt{9}} \\ &= \frac{40}{45} \\ &= \frac{8}{9} \end{aligned}$$

$$\therefore \quad \sin\theta = \sqrt{1 - \left(\tfrac{8}{9}\right)^2} = \tfrac{\sqrt{17}}{9}$$

α is the angle of rotation of AD.

$\therefore$ The angle between side AB and AD

$$\begin{aligned} &= \alpha + \theta \\ &= 90^\circ \quad \ldots . \text{[Given]} \end{aligned}$$

$$\begin{aligned} &\therefore \quad \cos(\alpha+\theta) = \cos(90^\circ) \\ &\therefore \quad \cos\alpha\cos\theta - \sin\alpha\sin\theta = 0 \\ &\therefore \quad 8\cos\alpha = \sqrt{17}\sin\alpha \\ &\therefore \quad 64\cos^2\alpha = 17\left(1-\cos^2\alpha\right) \\ &\therefore \quad 81\cos^2\alpha = 17 \\ &\therefore \quad \cos\alpha = \frac{\sqrt{17}}{9} \end{aligned}$$

Question 4

The values of a and b, so that the function

$$f(x) = \begin{cases} x + a\sqrt{2}\sin x, 0 \le x \le \frac{\pi}{4} \\ 2x\cot x + b, \frac{\pi}{4} \le x \le \frac{\pi}{2} \\ a\cos 2x - b\sin x, \frac{\pi}{2} < x \le \pi \end{cases}$$

is continuous for $0 \leq x \leq \pi$, are respectively given by

Options:

A. $-\frac{\pi}{12}, \frac{\pi}{6}$

B. $-\frac{\pi}{6}, -\frac{\pi}{12}$

C. $\frac{\pi}{6}, \frac{\pi}{12}$

D. $\frac{\pi}{6}, -\frac{\pi}{12}$

Answer: D

Solution:

As the given function is continuous at $x = \frac{\pi}{4}$ and $\frac{\pi}{2}$, we get

$$\lim_{x \to \frac{\pi^-}{4}} \mathrm{f}(x) = \lim_{x \to \frac{\pi^+}{4}} \mathrm{f}(x)$$

$$\therefore \quad \lim_{x \to \frac{\pi}{4}} (x + \mathrm{a}\sqrt{2} \sin x) = \lim_{x \to \frac{\pi}{4}} (2x \cot x + \mathrm{b})$$

$$\therefore \quad \frac{\pi}{4} + \mathrm{a} = \frac{2\pi}{4} + \mathrm{b}$$

$$\therefore \quad \mathrm{a} - \mathrm{b} = \frac{\pi}{4} \quad \text{... (i)}$$

Also, $\lim_{x \to \frac{\pi^-}{2}} \mathrm{f}(x) = \lim_{x \to \frac{\pi^+}{2}} \mathrm{f}(x)$

$$\therefore \quad \lim_{x \to \frac{\pi}{2}} 2x \cot x + b = \lim_{x \to \frac{\pi}{2}} a \cos 2x - b \sin x$$

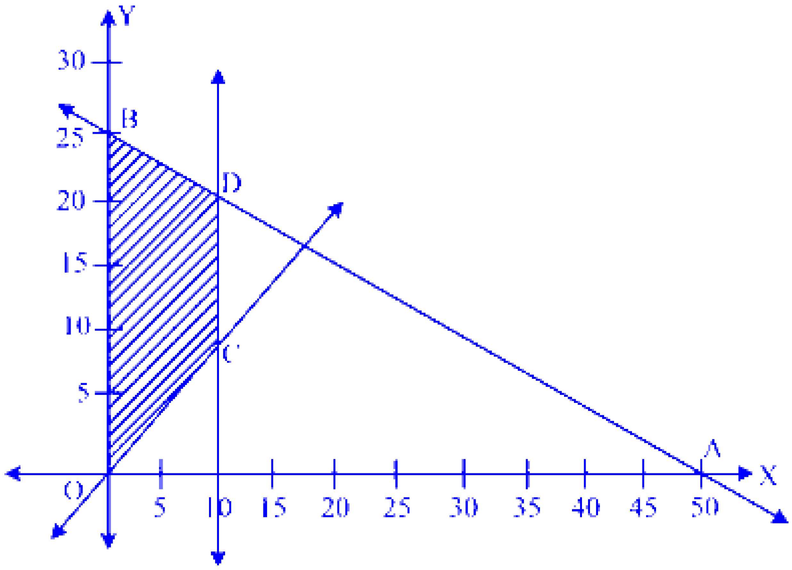
$$\therefore \quad 0 + b = -a - b$$

$$\therefore \quad a + 2b = 0 \quad \text{... (ii)}$$

Solving equations (i) and (ii), we get $a = \frac{\pi}{6}$ and $b = \frac{-\pi}{12}$

Question 5

For a feasible region OCDBO given below, the maximum value of the objective function $z = 3x + 4y$ is



Options:

A. 70

B. 100

C. 110

D. 130

Answer: C

Solution:

Corner points of the given feasible region are $O(0,0), C(10,10), D(10,20), B(0,25)$

$\therefore$ z at $C(10,10) = 70$,

z at $D(10,20) = 110$,

z at $B(0,25) = 100$

$\therefore$ The maximum value of z is 110.

Question 6

If $g(x) = 1+\sqrt{x}$ and $f(g(x)) = 3+2\sqrt{x}+x$ then $f(f(x))$ is

Options:

A. $x^2 + 4x + 6$

B. $x^4 + x^2 + 6$

C. $x^2 + x + 6$

D. $x^4 + 4x^2 + 6$

Answer: D

Solution:

$$
\begin{aligned}
& \mathrm{g}(x)=1+\sqrt{x} \text{ and } \mathrm{f}(\mathrm{g}(x))=3+2\sqrt{x}+x \\
\therefore \quad & \mathrm{f}(\mathrm{g}(x))=\left[(\sqrt{x})^2+2\sqrt{x}+1\right]+2 \\
& =(\sqrt{x}+1)^2+2 \\
& =[\mathrm{g}(x)]^2+2 \\
& \Rightarrow \mathrm{f}(x)=x^2+2 \\
& \Rightarrow \mathrm{f}(\mathrm{f}(x))=\left(x^2+2\right)^2+2=x^4+4x^2+6
\end{aligned}
$$

Question 7

The approximate value of $\sin\left(60^{\circ}0'10''\right)$ is (given that $\sqrt{3}=1.732, 1^{\circ}=0.0175^{\circ}$)

Options:

A. 0.08660243

B. 0.0008660243

C. 0.8660243

D. 0.008660243

Answer: C

Solution:

Let $\mathrm{f}(x)=\sin x$

$\therefore \quad \mathrm{f}'(x)=\cos x$

Here, $a = 60°$ and

$$h = 10'' = \left(\frac{1}{360}\right)^{\circ} = \frac{1}{360} \times 0.0175^c = 0.000049^c$$

$$f(a) = \sin(60°) = \frac{\sqrt{3}}{2} = \frac{1.732}{2} = 0.866$$

$$f'(a) = \cos(60) = \frac{1}{2} = 0.5$$

$$\therefore \quad f(a+h) \approx f(a) + hf'(a)$$

$$\therefore \quad \sin\left(60°0'10'' \approx 0.866 + 0.000049 \times 0.5\right.$$

$$\approx 0.866024$$

Question 8

The decay rate of radio active material at any time t is proportional to its mass at that time. The mass is 27 grams when $t = 0$. After three hours it was found that 8 grams are left. Then the substance left after one more hour is

Options:

A. $\frac{27}{8}$ grams

B. $\frac{81}{4}$ grams

C. $\frac{16}{3}$ grams

D. $\frac{16}{9}$ grams

Answer: C

Solution:

Let 'x' be the mass of the material at time 't'.

$\therefore \quad \frac{dx}{dt} = -kx$, ($-$ve sign indicates decay.)

$\therefore \quad \int \frac{dx}{x} = -k \int dt$

$\therefore \quad \log|x| = -kt + c$

When $t = 0, x = 27$

$\therefore \quad c = \log 27$

$\therefore \quad \log |x| = -kt + \log 27$

When $t = 3, x = 8$

$\therefore \quad k = \log\left(\frac{3}{2}\right)$

When $t = 4$, we get

$$\log |x| = -4 \log\left(\frac{3}{2}\right) + \log 27$$
$$\therefore \quad \log |x| = \log\left(\frac{16}{3}\right)$$
$$\therefore \quad x = \frac{16}{3} \text{ grams}$$

Question 9

The derivative of $f(\tan x)$ w.r.t. $g(\sec x)$ at $x = \frac{\pi}{4}$ where $f'(1) = 2$ and $g'(\sqrt{2}) = 4$ is

Options:

A. $\frac{1}{\sqrt{2}}$

B. $\sqrt{2}$

C. 1

D. 0

Answer: A

Solution:

Let $\mathrm{p} = \mathrm{f}(\tan x)$ and $\mathrm{q} = \mathrm{g}(\sec x)$

$$\therefore \quad \frac{\mathrm{dp}}{\mathrm{d}x} = \mathrm{f}'(\tan x) \times \sec^2 x \text{ and}$$

$$\frac{\mathrm{dq}}{\mathrm{d}x} = \mathrm{g}'(\sec x) \times \sec x \tan x$$

$$\therefore \quad \left.\frac{\mathrm{dp}}{\mathrm{dx}}\right|_{x=\frac{\pi}{4}} = \mathrm{f}'(1) \times 2 = 4,$$

$$\left.\frac{\mathrm{dq}}{\mathrm{d}x}\right|_{x=\frac{\pi}{4}} = \mathrm{g}'(\sqrt{2}) \times \sqrt{2} = 4\sqrt{2}$$

$$\therefore \quad \text{Required Derivative } = \left(\frac{\left.\frac{\mathrm{dp}}{\mathrm{d}x}\right|_{x=\frac{\pi}{4}}}{\left.\frac{\mathrm{dq}}{\mathrm{d}x}\right|_{x=\frac{\pi}{4}}}\right) = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Question 10

The p.m.f of random variate X is

$$P(X) = \begin{cases} \frac{2x}{\mathrm{n}(\mathrm{n}+1)}, & x = 1, 2, 3, \ldots\ldots, \mathrm{n} \\ 0, & \text{otherwise} \end{cases}$$

Then $\mathrm{E}(\mathrm{X}) =$

Options:

A. $\frac{\mathrm{n}+1}{3}$

B. $\frac{2\mathrm{n}+1}{3}$

C. $\frac{\mathrm{n}+2}{3}$

D. $\frac{2n-1}{3}$

Answer: B

Solution:

$$P(X) = \begin{cases} \frac{2x}{n(n+1)}, x = 1, 2, 3, \ldots, n \\ 0 \quad , \text{otehwise} \end{cases}$$

$$\therefore \quad E(X) = \sum_{i=1}^{n} x_i p(x_i)$$

$$= \frac{2}{n(n+1)} + \frac{8}{n(n+1)} + \ldots + \frac{2n^2}{n(n+1)}$$

$$= \frac{2(1^2 + 2^2 + \ldots + n^2)}{n(n+1)}$$

$$= \frac{2n(n+1)(2n+1)}{6n(n+1)}$$

$$= \frac{2n+1}{3}$$

Question 11

The angle between the tangents to the curves $y = 2x^2$ and $x = 2y^2$ at $(1, 1)$ is

Options:

A. $\tan^{-1}\left(\frac{15}{8}\right)$

B. $\tan^{-1}\left(\frac{7}{8}\right)$

C. $\tan^{-1}\left(\frac{3}{4}\right)$

D. $\tan^{-1}\left(\frac{1}{4}\right)$

Answer: A

Solution:

$y = 2x^2$

$\therefore$ Slope of the tangent to this curve is

$$\frac{dy}{dx} = m_1 = 4x$$

$\therefore$ at $(1, 1), m_1 = 4$

$x = 2y^2$

$\therefore$ Slope of the tangent to this curve is $\frac{dy}{dx} = m_2 = \frac{1}{4y}$

$\therefore \quad$ at $(1,1), m_2 = \frac{1}{4}$

Let θ be the angle between two tangents.

$$\therefore \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{4 - \frac{1}{4}}{1 + 4 \times \frac{1}{4}}\right| = \frac{15}{8}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{15}{8}\right)$$

Question 12

If the area of the triangle with vertices $(1,2,0)$, $(1,0,2)$ and $(0,x,1)$ is $\sqrt{6}$ square units, then the value of x is

Options:

A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

Let $A \equiv (1,2,0), B \equiv (1,0,2)$ and $C \equiv (0,x,1)$

$\therefore \quad \overline{AB} = -2\hat{j} + 2\hat{k}$ and $\overline{AC} = -\hat{i} + (x-2)\hat{j} + \hat{k}$

$\therefore \quad$ Area of $\triangle ABC = \frac{1}{2}|\overline{AB} \times \overline{AC}| = \sqrt{6}$

$$|\overline{AB} \times \overline{AC}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -2 & 2 \\ -1 & x-2 & 1 \end{vmatrix}$$
$$= \hat{i}[-2 - 2(x-2)] - \hat{j}(0+2) + \hat{k}(0-2)$$
$$= (2-2x)\hat{i} - 2\hat{j} - 2\hat{k}$$

$$\therefore \quad \frac{1}{2}|\overline{AB} \times \overline{AC}| = \sqrt{6}$$

$$\Rightarrow \frac{1}{2}\sqrt{(2-2x)^2+4+4} = \sqrt{6}$$

$$\Rightarrow (2-2x)^2 = 16$$

$$\Rightarrow 4 - 8x + 4x^2 = 16$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x-3)(x+1) = 0$$

$$\Rightarrow x = 3 \text{ or } -1$$

Question 13

An experiment succeeds twice as often as it fails. Then the probability, that in the next 6 trials there will be atleast 4 successes, is

Options:

A. $\frac{1}{729}$

B. $\frac{496}{729}$

C. $\frac{233}{729}$

D. $\frac{491}{729}$

Answer: B

Solution:

Experiment succeeds twice as often as it fails.

$\therefore$ According to the given condition, if 'p' is success and 'q' is failure, then p = 2q

$$\therefore \quad \mathrm{p}+\mathrm{q}=1 \Rightarrow 2\mathrm{q}+\mathrm{q}=1$$

$$\Rightarrow \mathrm{q}=\frac{1}{3} \text{ and } \mathrm{p}=\frac{2}{3}$$

Here, $\mathrm{n}=6$

Let X be the random variable

$\therefore \quad X \sim B(n,p)$

$\therefore \quad$ Required probability

$= \mathrm{P}(\mathrm{X} \geq 4)$

$= \mathrm{P}(\mathrm{X}=4)+\mathrm{P}(\mathrm{X}=5)+\mathrm{P}(\mathrm{X}=6)$

$= {}^6\mathrm{C}_4\mathrm{p}^4\mathrm{q}^2+{}^6\mathrm{C}_5\mathrm{p}^5\mathrm{q}+{}^6\mathrm{C}_6\mathrm{p}^6$

$= 15\times\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2+6\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)+\left(\frac{2}{3}\right)^6$

$= \frac{496}{729}$

Question 14

The co-ordinates of the points on the line $2x - y = 5$ which are the distance of 1 unit from the line $3x + 4y = 5$ are

Options:

A. $\left(\frac{30}{11}, \frac{-5}{11}\right), \left(\frac{20}{11}, \frac{15}{11}\right)$

B. $\left(\frac{-30}{11}, \frac{5}{11}\right), \left(\frac{-20}{11}, \frac{15}{11}\right)$

C. $\left(\frac{30}{11}, \frac{5}{11}\right), \left(\frac{20}{11}, \frac{-15}{11}\right)$

D. $\left(\frac{-30}{11}, \frac{5}{11}\right), \left(\frac{-20}{11}, \frac{-15}{11}\right)$

Answer: C

Solution:

Let (x_1, y_1) be the required point

$\therefore \quad 2x_1 - y_1 = 5$ ..... (i)

Also, (x_1, y_1) is at the distance of 1 unit from line $3x + 4y = 5$

$\therefore \quad 1 = \left|\frac{3x_1+4y_1-5}{\sqrt{9+16}}\right|$

$\therefore \quad \pm 5 = 3x_1 + 4y_1 - 5$

$\therefore \quad 3x_1 + 4y_1 - 5 = 5 \quad$ or $3x_1 + 4y_1 - 5 = -5$

$\therefore \quad 3x_1 + 4y_1 = 10 \quad$ .... (ii)

or

$3x_1 + 4y_1 = 0$ .... (iii)

Solving equations (i) and (ii), we get

$x_1 = \frac{30}{11}$ and $y_1 = \frac{5}{11}$

Solving equation (i) and (iii), we get

$x_1 = \frac{20}{11}$ and $y_1 = \frac{-15}{11}$

$\therefore \left(\frac{30}{11}, \frac{5}{11}\right)$ and $\left(\frac{20}{11}, \frac{-15}{11}\right)$ are the required points.

Question 15

If $x = \operatorname{cosec}\left(\tan^{-1}\left(\cos\left(\cot^{-1}\left(\sec\left(\sin^{-1} a\right)\right)\right)\right)\right), \mathrm{a} \in [0, 1]$

Options:

A. $x^2 - \mathrm{a}^2 = 3$

B. $x^2 + \mathrm{a}^2 = 3$

C. $x^2 - \mathrm{a}^2 = 2$

D. $x^2 + \mathrm{a}^2 = 2$

Answer: B

Solution:

$$\begin{aligned}
x &= \operatorname{cosec}\left(\tan^{-1}\left(\cos\left(\cot^{-1}\left(\sec\left(\sin^{-1} \mathrm{a}\right)\right)\right)\right)\right) \\
&= \operatorname{cosec}\left(\tan^{-1}\left(\cos\left(\cot^{-1}\left(\sec\left(\sec^{-1} \frac{1}{\sqrt{1-\mathrm{a}^2}}\right)\right)\right)\right)\right) \\
&= \operatorname{cosec}\left(\tan^{-1}\left(\cos\left(\cot^{-1}\left(\frac{1}{\sqrt{1-\mathrm{a}^2}}\right)\right)\right)\right) \\
&= \operatorname{cosec}\left(\tan^{-1}\left(\cos\left(\cos^{-1} \frac{1}{\sqrt{2-\mathrm{a}^2}}\right)\right)\right) \\
&= \operatorname{cosec}\left(\tan^{-1}\left(\frac{1}{\sqrt{2-\mathrm{a}^2}}\right)\right) \\
&= \operatorname{cosec}\left(\operatorname{cosec}^{-1}\left(\sqrt{3-\mathrm{a}^2}\right)\right) \\
\therefore \quad x &= \sqrt{3-\mathrm{a}^2} \\
\therefore \quad x^2 + \mathrm{a}^2 &= 3
\end{aligned}$$

Question 16

Let $\overline{\mathrm{A}}$ be a vector parallel to line of intersection of planes P_1 and P_2 through origin. P_1 is parallel to the vectors $2\hat{j}+3\hat{k}$ and $4\hat{j}-3\hat{k}$ and P_2 is parallel to $\hat{j}-\hat{k}$ and $3\hat{i}+3\hat{j}$, then the angle between $\bar{A}$ and $2\hat{i}+\hat{j}-2\hat{k}$ is

Options:

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{6}$

D. $\frac{3\pi}{4}$

Answer: D

Solution:

Vector equation of the plane passing through the point $A(\bar{a})$ and parallel to non-zero vectors $\bar{b}$ and $\overline{\mathrm{c}}$ is $\overline{\mathrm{r}}\cdot(\overline{\mathrm{b}}\times\overline{\mathrm{c}})=\overline{\mathrm{a}}\cdot(\overline{\mathrm{b}}\times\overline{\mathrm{c}})$

Plane P_1 is passing through the origin and parallel to vectors $\overline{b_1}=2\hat{j}+3\hat{k}$ and $\overline{c_1}=4\hat{j}-3\hat{k}$

$$\therefore \quad \overline{b_1}\times\overline{c_1}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 2 & 3\\ 0 & 4 & -3\end{vmatrix}=-18\hat{i}$$

$\therefore$ Equation of P_1 is: $r\cdot(-18\mathrm{i})=0$

Plane P_2 is passing through the origin and parallel to vectors $\overline{b_2}=\hat{j}-\hat{k}$ and $\overline{c_2}=3\hat{i}+3\hat{j}$

$$\therefore \quad \overline{b_2}\times\overline{c_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & -1\\ 3 & 3 & 0\end{vmatrix}=3\hat{i}-3\hat{j}-3\hat{k}$$

$\therefore$ Equation of P_2 is : $r_2\cdot(3\hat{i}-3\hat{j}-3\hat{k})=0$

Note that $\overline{\mathrm{A}}$ is parallel to the cross product of $-18\hat{i}$ and $3\hat{i}-3\hat{j}-3\hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -18 & 0 & 0 \\ 3 & -3 & -3 \end{vmatrix} = -54\hat{j} + 54\hat{k}$$

Let θ be the required angle.

$\therefore \quad \theta =$ Angle between $54(-\hat{j} + \hat{k})$ and $2\hat{i} + \hat{j} - 2\hat{k}$

$$\therefore \quad \cos\theta = \frac{54 \times (-1-2)}{54\sqrt{0+1+1}\sqrt{4+1+4}}$$
$$= \pm\frac{3}{3\sqrt{2}}$$
$$= \pm\frac{1}{\sqrt{2}}$$

$$\therefore \quad \theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

Question 17

$$\int \frac{\operatorname{cosec} x dx}{\cos^2\left(1+\log\tan\frac{x}{2}\right)} =$$

Options:

A. $\tan\left(1+\log\left(\tan\frac{x}{2}\right)\right) + \mathrm{c}$, where c is constant of integration

B. $\tan(1+\log(\tan x)) + c$, where c is constant of integration

C. $\tan\left(\log\left(\tan\frac{x}{2}\right)\right) + c$, where c is constant of integration.

D. $\tan\left(\tan\frac{x}{2}\right) + c$, where c is constant of integration.

Answer: A

Solution:

Let $I = \int \frac{\operatorname{cosec} x dx}{\cos^2\left(1+\log\tan\frac{x}{2}\right)} dx$

Let $1 + \log\left(\tan\frac{x}{2}\right) = \mathrm{t}$

Differentiating both sides w.r.t. t, we get

$\frac{1}{\tan\frac{x}{2}} \sec^2\frac{x}{2} \times \frac{1}{2} \, \mathrm{d}x = \mathrm{dt}$

$\therefore \quad \frac{\frac{1}{2 \sin \frac{x}{2} \cos \frac{x}{2}}}{} dx = dt$

$\therefore \quad \operatorname{cosec} x \, dx = dt$

$$\therefore \quad I = \int \frac{1}{\cos^2 t} dt = \int \sec^2 t dt$$
$$= \tan(t) + c$$
$$= \tan\left(1 + \log\left(\tan \frac{x}{2}\right)\right) + c$$

Question 18

If the variance of the numbers $-1, 0, 1, k$ is 5, where $k > 0$, then k is equal to

Options:

A. $2\sqrt{\frac{10}{3}}$

B. $2\sqrt{6}$

C. $4\sqrt{\frac{5}{3}}$

D. $\sqrt{6}$

Answer: B

Solution:

$$\text{Variance } = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2$$

Here, n = 4 and variance = 5

$$\therefore \quad 5 = \frac{1}{4}\left[(-1)^2 + (0)^2 + (1)^2 + k^2\right] - \left(\frac{-1+0+1+k}{4}\right)^2$$

$$\therefore \quad 5 = \frac{2+k^2}{4} - \frac{k^2}{16}$$

$$\therefore \quad 80 = 8 + 4k^2 - k^2$$

$$\therefore \quad 3k^2 = 72$$

$$\therefore \quad k^2 = 24$$

$$\therefore \quad k = 2\sqrt{6} \quad \ldots [\because k > 0]$$

Question 19

The differential equation $\cos(x+y)\mathrm{d}y = \mathrm{d}x$ has the general solution given by

Options:

A. $y = \sin(x+y) + c$, where c is a constant.

B. $y = \tan(x+y) + c$, where c is a constant

C. $y = \tan\left(\frac{x+y}{2}\right) + c$, where c is a constant

D. $y = \frac{1}{2}\tan(x+y) + c$, where c is a constant

Answer: C

Solution:

$$\cos(x+y)\mathrm{d}y = \mathrm{d}x$$

$$\therefore \quad \frac{\mathrm{d}x}{\mathrm{d}y} = \cos(x+y) \quad \text{..... (i)}$$

Put $x + y = u$ .... (ii)

Differentiating w.r.t. y, we get

$\frac{\mathrm{d}x}{\mathrm{d}y} + 1 = \frac{\mathrm{du}}{\mathrm{d}y}$

$\therefore \quad \frac{dx}{dy} = \frac{du}{dy} - 1$ ..... (iii)

Substituting (ii) and (iii) in (i), we get

$$\frac{du}{dy} - 1 = \cos u$$

$$\therefore \quad \frac{du}{1 + \cos u} = dy$$

$$\therefore \quad \frac{du}{2\cos^2\left(\frac{u}{2}\right)} = dy$$

Integrating on both sides, we get

$$\frac{1}{2}\int \sec^2\left(\frac{u}{2}\right) du = \int dy$$

$$\therefore \quad y = \tan\left(\frac{x+y}{2}\right) + c$$

Question 20

The area of the region bounded by the curves $y = e^x, y = \log x$ and lines $x = 1, x = 2$ is

Options:

A. $(e-1)^2$ sq. units

B. $\left(e^2 - e + 1\right)$ sq. units

C. $\left(e^2 - e + 1 - 2\log 2\right)$ sq. units

D. $\left(e^2 + e - 2\log 2\right)$ sq. units

Answer: C

Solution:

Required Area

$$= \int_1^2 (e^x - \log x)dx$$

$$= [e^x]_1^2 - \int_1^2 1 \log x \, dx$$

$$= (e^2 - e) - \left[x \log x - \int_1^2 1 \, dx\right]$$

$$= (e^2 - e) - [x \log x - x]_1^2$$

$$= (e^2 - e) - [(2 \log 2 - 2) - (1 \log 1 - 1)]$$

$$= e^2 - e - (2 \log 2 - 2 - 0 + 1)$$

$$= e^2 - e - (2 \log 2 - 1)$$

$$= \left(e^2 - e + 1 - 2 \log 2\right) \text{ sq. units}$$

Question 21

If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha \log x + \beta x^2 + x, \alpha$ and β are constants, then the value of $\alpha^2 + 2\beta$ is

Options:

A. -3

B. 3

C. $\frac{3}{2}$

D. 5

Answer: B

Solution:

According to the given condition,

$$f'(1) = 0 \text{ and } f'(2) = 0$$

$$f(x) = \alpha \log x + \beta x^2 + x$$

$$\therefore \quad f'(x) = \frac{\alpha}{x} + 2\beta x + 1$$

$$\therefore \quad f'(-1) = 0 \Rightarrow \alpha + 2\beta = 1 \quad \text{.... (i)}$$

$$\text{and } f'(2) = 0 \Rightarrow \alpha + 8\beta = -2 \quad \text{.... (ii)}$$

$\therefore$ From (i) and (ii), we get

$$\beta=\frac{-1}{2} \text{ and } \alpha=2$$

$\therefore$ $\alpha^2+2\beta=4-1=3$

Question 22

A plane is parallel to two lines whose direction ratios are $1,0,-1$ and $-1,1,0$ and it contains the point $(1,1,1)$. If it cuts the co-ordinate axes at $\mathrm{A}, \mathrm{B}, \mathrm{C}$, then the volume of the tetrahedron OABC (in cubic units) is

Options:

A. $\frac{9}{4}$

B. $\frac{9}{2}$

C. 9

D. 27

Answer: B

Solution:

Equation of the plane passing through $(1,1,1)$ is given as

$\mathrm{a}(x-1)+\mathrm{b}(y-1)+\mathrm{c}(\mathrm{z}-1)=0$ .... (i)

As the plane is parallel to the lines having direction ratios $1,0,-1$ and $-1,1,0$, we get $\mathrm{a}-\mathrm{c}=0$ and $-\mathrm{a}+\mathrm{b}=0$

$\Rightarrow \mathrm{a}=\mathrm{b}=\mathrm{c}$ ..... (ii)

$\therefore$ From (i) and (ii), we get

$$x-1+y-1+z-1=0$$
$$\therefore x+y+z=3 \quad \Rightarrow \frac{x}{3}+\frac{y}{3}+\frac{z}{3}=1$$

$\therefore$ Co-ordinates of A, B, C are $(3,0,0),(0,3,0)$ and $(0,0,3)$ respectively.

$\therefore$ Volume of tetrahedron OABC

$$= \frac{1}{6}\begin{vmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{vmatrix}$$
$$= \frac{1}{6} \times 27$$
$$= \frac{9}{2} \text{ cu. units}$$

Question 23

The function $\mathrm{f}(x) = \sin^4 x + \cos^4 x$ is increasing in

Options:

A. $0 < x < \frac{\pi}{8}$

B. $\frac{\pi}{4} < x < \frac{\pi}{2}$

C. $\frac{3\pi}{8} < x < \frac{5\pi}{8}$

D. $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

Answer: B

Solution:

$$\mathrm{f}(x) = \sin^4 x + \cos^4 x$$
$$\therefore \quad \mathrm{f}'(x) = 4\sin^3 x \cos x - 4\cos^3 x \sin x$$
$$= 4\sin x \cos x \left(\sin^2 x - \cos^2 x\right)$$
$$= -2\sin 2x \cos 2x$$
$$= -\sin 4x$$

$\therefore$ If $\mathrm{f}(x)$ is increasing, then $\mathrm{f}'(x) > 0$

i.e., $-\sin 4x > 0 \Rightarrow \pi < 4x < 2\pi$
$$\Rightarrow \frac{\pi}{4} < x < \frac{\pi}{2}$$

Question 24

If $a > 0$ and $z = \frac{(1+i)^2}{a+i}$, $(i = \sqrt{-1})$ has magnitude $\frac{2}{\sqrt{5}}$, then $\bar{z}$ is equal to

Options:

A. $-\frac{2}{5} + \frac{4}{5}\mathrm{i}$

B. $\frac{2}{5} - \frac{4}{5}\mathrm{i}$

C. $-\frac{2}{5} - \frac{4}{5}\mathrm{i}$

D. $\frac{2}{5} + \frac{4}{5}\mathrm{i}$

Answer: B

Solution:

$$z = \frac{(1+i)^2}{a+i} = \frac{2i}{a+i} = \frac{2i(a-i)}{(a+i)(a-i)} = \frac{2+2ai}{a^2+1} \quad \text{.... (i)}$$

$$|z| = \frac{2}{\sqrt{5}} \Rightarrow \frac{4}{(a^2+1)^2} + \frac{4a^2}{(a^2+1)^2} = \frac{4}{5}$$

$$\Rightarrow 20 + 20a^2 = 4\left(a^4 + 2a^2 + 1\right)$$

$$\Rightarrow 4a^4 - 12a^2 - 16 = 0$$

$$\Rightarrow a^4 - 3a^2 - 4 = 0$$

$$\Rightarrow \left(a^2 - 4\right)\left(a^2 + 1\right) = 0$$

$$\Rightarrow a^2 = 4 \text{ and } a^2 = -1$$

$$\Rightarrow a = 2 \quad \ldots [\because a > 0]$$

$$\therefore \quad \text{(i)} \Rightarrow z = \frac{2}{5} + \frac{4}{5}i$$

$$\therefore \quad \bar{z} = \frac{2}{5} - \frac{4}{5}i$$

Question 25

The integral $\int \frac{\sin^2 x \cos^2 x}{\left(\sin^5 x+\cos^3 x \sin^2 x+\sin^3 x \cos^2 x+\cos^5 x\right)^2} \mathrm{d}x$ is equal to

Options:

A. $\frac{1}{3\left(1+\tan^3 x\right)}+\mathrm{c}$, where c is a constant of integration.

B. $\frac{-1}{3\left(1+\tan^3 x\right)}+\mathrm{c}$, where c is a constant of integration.

C. $\frac{1}{1+\cot^3 x}+\mathrm{c}$, where c is a constant of integration.

D. $\frac{-1}{1+\cos^3 x}+\mathrm{c}$, where c is a constant of integration.

Answer: B

Solution:

Let

$$
\begin{aligned}
I &= \int \frac{\sin^2 x \cos^2 x}{\left(\sin^5 x+\cos^3 x \sin^2 x+\sin^3 x \cos^2 x+\cos^5 x\right)^2} \mathrm{d}x \\
&= \int \frac{\sin^2 x \cos^2 x}{\left(\sin^5 x+\sin^3 x \cos^2 x+\cos^3 x \sin^2 x+\cos^5 x\right)^2} \mathrm{d}x \\
&= \int \frac{\sin^2 x \cos^2 x}{\left[\sin^3 x\left(\sin^2 x+\cos^2 x\right)+\cos^3 x\left(\sin^2 x+\cos^2 x\right)\right]^2} \mathrm{d}x \\
&= \int \frac{\sin^2 x \cos^2 x}{\left(\sin^3 x+\cos^3 x\right)^2} \mathrm{d}x \\
&= \int \frac{\sec^2 x \tan^2 x}{\left(1+\tan^3 x\right)^2} \mathrm{d}x
\end{aligned}
$$

....[Dividing numerator and denominator by $\cos^6 x$]

Let $1+\tan^3 x=\mathrm{t}$

Differentiating w.r.t. x, we get

$$
\begin{aligned}
& 3 \tan^2 x \sec^2 x \,\mathrm{d}x=\mathrm{dt} \\
\therefore \quad & \tan^2 x \sec^2 x \,\mathrm{d}x=\frac{1}{3} \mathrm{dt}
\end{aligned}
$$

$$
\begin{aligned}
\therefore \quad I &= \frac{1}{3}\int \frac{1}{t^2}dt \\
&= \frac{-1}{3t} + c \\
&= \frac{-1}{3\left(1+\tan^3 x\right)} + c
\end{aligned}
$$

Question 26

The equation of the plane through $(-1, 1, 2)$ whose normal makes equal acute angles with co-ordinate axes is

Options:

A. $x + y + z - 3 = 0$

B. $x + y + z - 2 = 0$

C. $x + y - z - 2 = 0$

D. $x - y + z - 3 = 0$

Answer: B

Solution:

Note that $(-1, 1, 2)$ is satisfied by only option (B)

Alternate Method:

Let A $\equiv (-1, 1, 2)$

$$
\begin{aligned}
\therefore \quad \bar{a} &= -\hat{i} + \hat{j} + 2\hat{k} \\
\bar{n} &= \hat{i} + \hat{j} + \hat{k}
\end{aligned}
$$

$\therefore$ equation of plane is $\bar{r} \cdot \bar{n} = \bar{a} \cdot \bar{n}$

$$
\begin{aligned}
&\Rightarrow \bar{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = (-\hat{i} + \hat{j} + 2\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) \\
&\Rightarrow \bar{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2 \\
&\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 2 \\
&\Rightarrow x + y + z - 2 = 0
\end{aligned}
$$

Question 27

If $\mathrm{T_n}$ denotes the number of triangles which can be formed using the vertices of regular polygon of n sides and $T_{n+1} - T_n = 21$, then $\mathrm{n} =$

Options:

A. 5

B. 7

C. 6

D. 4

Answer: B

Solution:

According to the given condition, $\mathrm{T_n} = {}^{\mathrm{n}}\mathrm{C}_3$

$\therefore \quad \mathrm{T_{n+1}} - \mathrm{T_n} = 21 \Rightarrow {}^{\mathrm{n+1}}\mathrm{C}_3 - {}^{\mathrm{n}}\mathrm{C}_3 = 21$

Note that $\mathrm{n} = 7$ satisfies the above condition.

$\therefore \quad$ Option (B) is correct.

Question 28

Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Then the probability distribution of number of jacks is

Options:

A.

$\mathrm{X} = x$	0	1	2
$\mathrm{P}(\mathrm{X} = x)$	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

B.

$X = x$	0	1	2
$P(X = x)$	$\frac{1}{169}$	$\frac{144}{169}$	$\frac{24}{169}$

C.

$X = x$	0	1	2
$P(X = x)$	$\frac{24}{169}$	$\frac{1}{169}$	$\frac{144}{169}$

D.

$X = x$	0	1	2
$P(X = x)$	$\frac{144}{169}$	$\frac{1}{169}$	$\frac{24}{169}$

Answer: A

Solution:

Let X denotes the number of jacks

$\therefore$ Possible values of X are $0, 1, 2$

$\therefore \quad P(X = 0) = \frac{^{48}C_1 \times {}^{48}C_1}{^{52}C_1 \times {}^{52}C_1} = \frac{144}{169}$

$$P(X = 1) = \frac{^{48}C_1 \times {}^{4}C_1}{^{52}C_1 \times {}^{52}C_1} + \frac{^{4}C_1 \times {}^{48}C_1}{^{52}C_1 \times {}^{52}C_1} = \frac{24}{169}$$

$$P(X = 2) = \frac{^{4}C_1 \times {}^{4}C_1}{^{52}C_1 \times {}^{52}C_1} = \frac{1}{169}$$

$\therefore$ Option (A) is correct.

Question 29

If $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}, 0 \leq \alpha \leq \frac{\pi}{2}$, then the value of $\cos 2\theta$ is

Options:

A. $\cos 2\alpha$

B. $\sin\alpha$

C. $\cos\alpha$

D. $\sin 2\alpha$

Answer: D

Solution:

$$\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$$
$$\frac{\sin\theta}{\cos\theta} = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$$
$$\therefore \quad \sin\alpha\sin\theta + \cos\alpha\sin\theta = \sin\alpha\cos\theta - \cos\alpha\cos\theta$$
$$\therefore \quad \cos\alpha\cos\theta + \sin\alpha\sin\theta = \sin\alpha\cos\theta - \cos\alpha\sin\theta$$
$$\therefore \quad \cos(\alpha-\theta) = \sin(\alpha-\theta)$$
$$\therefore \quad \alpha-\theta = \frac{\pi}{4} \quad \ldots\left[\because 0 \le \alpha \le \frac{\pi}{2}\right]$$
$$\therefore \quad \theta = \alpha - \frac{\pi}{4}$$
$$\therefore \quad 2\theta = 2\alpha - \frac{\pi}{2}$$
$$\therefore \quad \cos 2\theta = \cos\left(2\alpha - \frac{\pi}{2}\right)$$
$$= \cos\left[-\left(\frac{\pi}{2} - 2\alpha\right)\right]$$
$$= \cos\left(\frac{\pi}{2} - 2\alpha\right) \quad \ldots[\because \cos(-\theta) = \cos\theta]$$
$$\therefore \quad \cos 2\theta = \sin 2\alpha$$

Question 30

The solution set of $8\cos^2\theta + 14\cos\theta + 5 = 0$, in the interval $[0, 2\pi]$, is

Options:

A. $\left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$

B. $\left\{\frac{\pi}{3}, \frac{4\pi}{3}\right\}$

C. $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

D. $\left\{\frac{2\pi}{3}, \frac{5\pi}{3}\right\}$

Answer: C

Solution:

$$
\begin{aligned}
& 8\cos^2\theta + 14\cos\theta + 5 = 0 \\
\therefore\quad & 8\cos^2\theta + 10\cos\theta + 4\cos\theta + 5 = 0 \\
\therefore\quad & 2\cos\theta(4\cos\theta + 5) + 1(4\cos\theta + 5) = 0 \\
\therefore\quad & (2\cos\theta + 1)(4\cos\theta + 5) = 0 \\
\therefore\quad & \cos\theta = \tfrac{-1}{2} \text{ or } \cos\theta = \tfrac{-5}{4}
\end{aligned}
$$

But $\cos\theta = \frac{-5}{4}$ is not possible as $\cos\theta \in [-1, 1]$ for all values of θ.

$$
\begin{aligned}
\therefore\quad & \cos\theta = \frac{-1}{2} \\
\therefore\quad & \theta \in \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}
\end{aligned}
$$

Question 31

A ladder 5 meters long rests against a vertical wall. If its top slides downwards at the rate of $10\ \mathrm{cm/s}$, then the angle between the ladder and the floor is decreasing at the rate of ________ rad./s when it's lower end is $4\ \mathrm{m}$ away from the wall.

Options:

A. -0.1

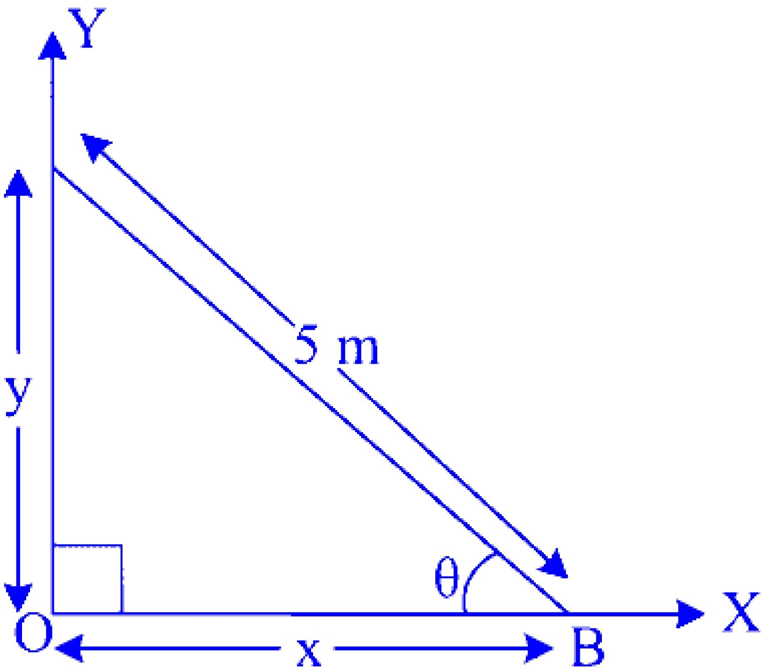
B. -0.025

C. 0.1

D. 0.025

Answer: D

Solution:



According to the figure, $x^2+y^2=25$ ..... (i)

Note that $\cos\theta=\frac{\mathrm{OB}}{\mathrm{AB}}=\frac{x}{5}$

$$\therefore \quad x=5\cos\theta$$
$$\therefore \quad \text{(i)} \Rightarrow 25\cos^2\theta+y^2=25$$

Differentiating w.r.t. 't', we get

$$-50\cos\theta\sin\theta\frac{\mathrm{d}\theta}{\mathrm{dt}}+2y\frac{\mathrm{d}y}{\mathrm{dt}}=0$$
$$25\sin\theta\cos\theta\frac{\mathrm{d}\theta}{\mathrm{dt}}=y\frac{\mathrm{d}y}{\mathrm{dt}}$$
$$\therefore \quad 25\sin\theta\cos\theta\frac{\mathrm{d}\theta}{\mathrm{dt}}=y(-0.1)$$
$$\ldots\left[\because \frac{\mathrm{d}y}{\mathrm{d}x}=-10\ \mathrm{cm/s}=-0.1\ \mathrm{m/s}\right]$$
$$\therefore \quad 25\sin\theta\cos\theta\frac{\mathrm{d}\theta}{\mathrm{dt}}=-(0.1)y \quad \ldots \text{(ii)}$$
$$\text{at } x=4, \cos\theta=\frac{4}{5}, \sin\theta=\frac{3}{5} \quad \text{and } y=3$$
$$\therefore \quad \text{(ii)} \Rightarrow 25\times\frac{3}{5}\times\frac{4}{5}\times\frac{\mathrm{d}\theta}{\mathrm{dt}}=-0.3$$
$$\Rightarrow \frac{\mathrm{d}\theta}{\mathrm{dt}}=-0.025$$

i.e., the angle is decreasing at the rate of $0.025\ \mathrm{rad/s}$

Question 32

If $\frac{dy}{dx} = y + 3$ and $y(0) = 2$, then $y(\log 2) =$

Options:

A. 5

B. 7

C. 13

D. -2

Answer: B

Solution:

$$\frac{dy}{dx} = y + 3$$
$$\Rightarrow \frac{dy}{y+3} = dx$$

Integrating on both sides, we get

$$\int \frac{dy}{y+3} = \int dx + c$$
$$\Rightarrow \log(y+3) = x + c \quad .... \text{(i)}$$

$y = 2$ when $x = 0$ ... [Given]

$\therefore \quad \log(2+3) = 0 + c \Rightarrow c = \log 5$

$\therefore \quad \log(y+3) = x + \log 5$ ... [From (i)]

$\Rightarrow y + 3 = 5e^x$

$\Rightarrow y = 5e^x - 3$

$\therefore \quad y(\log 2) = 5e^{\log 2} - 3 = 10 - 3$

$= 7$

Question 33

If $\log(x+y) = 2xy$, then $\frac{dy}{dx}$ at $x = 0$ is

Options:

A. 1

B. -1

C. 2

D. -2

Answer: A

Solution:

$\log(x+y)=2xy$ .... (i)

Differentiating w.r.t. x, we get

$$\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)=2y+2x\frac{dy}{dx}$$

$$\therefore \quad \frac{1}{x+y}+\frac{1}{x+y}\frac{dy}{dx}=2y+2x\frac{dy}{dx} \quad \text{.... (ii)}$$

$$\text{At } x=0,\ \text{(i)} \Rightarrow y=1$$

$$\therefore \quad \text{(ii)} \Rightarrow \left.\frac{dy}{dx}\right|_{x=0}=1$$

Question 34

If general solution of $\cos^2\theta-2\sin\theta+\frac{1}{4}=0$ is $\theta=\frac{n\pi}{A}+(-1)^n\frac{\pi}{B}, n\in Z$, then $A+B$ has the

Options:

A. 7

B. 6

C. 1

D. -7

Answer: A

Solution:

$$\cos^2\theta - 2\sin\theta + \frac{1}{4} = 0$$
$$\therefore \quad \left(1 - \sin^2\theta\right) - 2\sin\theta + \frac{1}{4} = 0$$
$$\therefore \quad \sin^2\theta + 2\sin\theta - \frac{5}{4} = 0$$
$$\therefore \quad 4\sin^2\theta + 8\sin\theta - 5 = 0$$
$$\therefore \quad 4\sin^2\theta + 10\sin\theta - 2\sin\theta - 5 = 0$$
$$\therefore \quad 2\sin\theta(2\sin\theta + 5) - 1(2\sin\theta + 5) = 0$$
$$\therefore \quad (2\sin\theta - 1)(2\sin\theta + 5) = 0$$
$$\therefore \quad \sin\theta = \frac{1}{2} \text{ or } \sin\theta = \frac{-5}{2}$$

But $\sin\theta = \frac{-5}{2}$ is not possible as $\sin\theta \in [-1, 1]$ for all values of θ.

$$\therefore \quad \sin\theta = \frac{1}{2}$$
$$\therefore \quad \sin\theta = \sin\frac{\pi}{6}$$
$$\therefore \quad \theta = \frac{n\pi}{1} + (-1)^n\frac{\pi}{6}$$
$$\therefore \quad A = 1 \text{ and } B = 6$$
$$\Rightarrow A + B = 7$$

Question 35

$\overline{\mathrm{u}}, \overline{\mathrm{v}}, \overline{\mathrm{w}}$ are three vectors such that $|\overline{\mathrm{u}}| = 1, |\bar{v}| = 2, |\bar{w}| = 3$. If the projection of $\bar{v}$ along $\bar{u}$ is equal to projection of $\bar{w}$ along $\bar{u}$ and $\bar{v}, \bar{w}$ are perpendicular to each other, then $|\bar{u} - \bar{v} + \bar{w}| =$

Options:

A. 4

B. $\sqrt{7}$

C. $\sqrt{14}$

D. 2

Answer: C

Solution:

$|\overline{\mathrm{u}}| = 1, |\overrightarrow{\overline{\mathrm{v}}}| = 2, |\overline{\mathrm{w}}| = 3$

According to the given condition, (Projection of $\overline{\mathrm{v}}$ along $\overline{\mathrm{u}}$) = (Projection of $\bar{w}$ along $\bar{u}$)

$\therefore \quad \frac{\overline{v} \cdot \overline{u}}{|\overline{u}|} = \frac{\overline{w} \cdot \overline{u}}{|\overline{u}|}$

$\therefore \quad \overline{v} \cdot \overline{u} = \overline{w} \cdot \overline{u}$

$\therefore \quad (\overline{w} - \overline{v}) \cdot \overline{u} = 0$ .... (i)

Now consider, $|\overline{u} - \overline{v} + \overline{w}| = \sqrt{|\overline{u} + \overline{w} - \overline{v}|^2}$

$= \sqrt{|\overline{u}|^2 + |\overline{w} - \overline{v}|^2 + 2\overline{u} \cdot (\overline{w} - \overline{v})}$

$= \sqrt{(1)^2 + |\overline{w} - \overline{v}|^2 + 0}$ .... [From (i)]

$= \sqrt{1 + |\overline{w}|^2 + |\overline{v}|^2 - 2(\overline{w} \cdot \overline{v})}$

$= \sqrt{1 + 9 + 4 + 0} \quad \ldots [\because \overline{w}$ and $\overline{v}$ are perpendicular]

$= \sqrt{14}$

Question 36

The distance of the point $P(-2, 4, -5)$ from the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is

Options:

A. $\frac{\sqrt{37}}{10}$

B. $\sqrt{\frac{37}{10}}$

C. $\frac{37}{\sqrt{10}}$

D. $\frac{37}{10}$

Answer: B

Solution:

Since the point is $(-2, 4, -5)$,

$\therefore \quad a = -2, \ b = 4, c = -5$

Given equation of line is

$\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$

$x_1 = -3, y_1 = 4, z_1 = -8$

d.r.s of the line are $3, 5, 6$

d.c.s are $\frac{3}{\sqrt{70}}, \frac{5}{\sqrt{70}}, \frac{6}{\sqrt{70}}$

Perpendicular distance of point from the line is

$$\sqrt{\begin{array}{c}\left[(\mathrm{a}-x_1)^2+(\mathrm{b}-y_1)^2+(\mathrm{c}-\mathrm{z}_1)^2\right]\\ -[(\mathrm{a}-x_1)l+(\mathrm{b}-y_1)\mathrm{m}+(\mathrm{c}-\mathrm{z}_1)\mathrm{n}]^2\end{array}}$$

$$= \sqrt{1^2+0+3^2-\left[\frac{1(3)}{\sqrt{70}}+\frac{0(5)}{\sqrt{70}}+\frac{3(6)}{\sqrt{70}}\right]^2}$$

$$= \sqrt{1+9-\left(\frac{3}{\sqrt{70}}+\frac{18}{\sqrt{70}}\right)^2}$$

$$= \sqrt{\frac{37}{10}}.\text{units}$$

Question 37

A and B are independent events with $\mathrm{P(A)} = \frac{1}{4}$ and $\mathrm{P(A \cup B) = 2P(B) - P(A)}$, then $\mathrm{P(B)}$ is

Options:

A. $\frac{1}{4}$

B. $\frac{3}{5}$

C. $\frac{2}{3}$

D. $\frac{2}{5}$

Answer: D

Solution:

$$\begin{aligned} & \mathrm{P(A \cup B) = 2P(B) - P(A)} \\ \therefore\ & \mathrm{P(A) + P(B) - P(A \cap B) = 2P(B) - P(A)} \\ \therefore\ & \mathrm{P(A) + P(B) - P(A) \cdot P(B) = 2P(B) - P(A)} \quad \ldots.[\because \text{ A and B are independent events}] \end{aligned}$$

$$\therefore \quad P(B) + P(A) \cdot P(B) = 2P(A)$$

$$\therefore \quad P(B) = \frac{2P(A)}{(1 + P(A))} = \frac{2 \times \frac{1}{4}}{(1 + \frac{1}{4})} = \frac{2}{5}$$

Question 38

$\int \frac{x^2+1}{x(x^2-1)} \mathrm{d}x =$

Options:

A. $\log x\left(x^2-1\right)+\mathrm{c}$, where c is a constant of integration.

B. $\log\left(\frac{x^2-1}{x}\right)+\mathrm{c}$, where c is a constant of integration.

C. $\log\left(x^2-1\right)+\mathrm{c}$, where c is a constant of integration.

D. $\log\left(\frac{x^2+1}{x}\right)+\mathrm{c}$, where c is a constant of integration.

Answer: B

Solution:

$$\text{Let I} = \int \frac{x^2+1}{x\left(x^2-1\right)} dx$$

$$= \int \frac{\frac{x^2+1}{x^2}}{\frac{x^2-1}{x}} dx$$

$$\text{Let t} = \frac{x^2-1}{x} \Rightarrow dt = \frac{x^2+1}{x^2} dx$$

$$\therefore \quad I = \int \frac{1}{t} dt = \log(t) + c = \log\left(\frac{x^2-1}{x}\right) + c$$

Question 39

If the matrix $A = \begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix}$ and $A^{-1} = x\,A + y I$, when I is a unit matrix of order 2 , then the value of $2x + 3y$ is

Options:

A. $\frac{8}{11}$

B. $\frac{4}{11}$

C. $\frac{-8}{11}$

D. $\frac{-4}{11}$

Answer: B

Solution:

$$A = \begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix}$$
$$\therefore |A| = 11$$
$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix}$$
$A^{-1} = x\,A + y I$, we get
$$\begin{bmatrix} \frac{1}{11} & \frac{-2}{11} \\ \frac{5}{11} & \frac{1}{11} \end{bmatrix} = \begin{bmatrix} x & 2x \\ -5x & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix}$$
$$\therefore \quad \begin{bmatrix} \frac{1}{11} & \frac{-2}{11} \\ \frac{5}{11} & \frac{1}{11} \end{bmatrix} = \begin{bmatrix} x+y & 2x \\ -5x & x+y \end{bmatrix}$$
$$\Rightarrow \quad x = \frac{-1}{11} \text{ and } y = \frac{2}{11}$$
$$\therefore \quad 2x + 3y = 2\left(\frac{-1}{11}\right) + 3\left(\frac{2}{11}\right) = \frac{4}{11}$$

Question 40

The inverse of the statement "If the surface area increase, then the pressure decreases.", is

Options:

A. If the surface area does not increase, then the pressure does not decrease.

B. If the pressure decreases, then the surface area increases.

C. If the pressure does not decreases, then the surface area does not increase.

D. If the surface area does not increase, then the pressure decreases.

Answer: A

Solution:

Let p : The surface area increases

q : The pressure decreases

Given statement is $p \rightarrow q$

$\therefore$ It's inverse is $\sim p \rightarrow \sim q$

$\therefore$ Option (A) is correct.

Question 41

In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is

Options:

A. $\frac{c}{3}$

B. $\frac{c}{\sqrt{3}}$

C. $\frac{3}{2}y$

D. $\frac{y}{\sqrt{3}}$

Answer: B

Solution:

Let a and b be the lengths of two sides of a triangle.

$\therefore$ According to the given condition,

$\mathrm{a}+\mathrm{b}=x$ and $\mathrm{ab}=y$

$$\therefore \quad x^2-c^2=y \Rightarrow (a+b)^2-c^2=ab$$
$$\Rightarrow a^2+b^2+2ab-c^2=ab$$
$$\Rightarrow a^2+b^2-c^2=-ab$$
$$\Rightarrow \frac{a^2+b^2-c^2}{2ab}=-\frac{1}{2}$$
$$\Rightarrow \cos \mathrm{C}=\frac{1}{2}$$
$$\Rightarrow \mathrm{C}=\frac{2\pi}{3}$$
$$\Rightarrow \text{circumradius} =\frac{c}{2\sin C}=\frac{c}{2\sin\left(\frac{2\pi}{3}\right)}=\frac{c}{\sqrt{3}}$$

Question 42

The contrapositive of "If x and y are integers such that xy is odd, then both x and y are odd" is

Options:

A. If both x and y are odd integers, then xy is odd.

B. If both x and y are even integers, then xy is even.

C. If x or y is an odd integer, then xy is odd.

D. If both x and y are not odd integers, then the product xy is not odd.

Answer: D

Solution:

Let p : x and y are integers such that xy is odd.

q : both x and y are odd.

$\therefore$ Given statement is $\mathrm{p} \rightarrow \mathrm{q}$

$\therefore$ Its contrapositive is $\sim \mathrm{q} \rightarrow \mathrm{p}$

$\therefore$ Option (D) is correct.

Question 43

$y=(1+x)\left(1+x^2\right)\left(1+x^4\right) \ldots \ldots \ldots\left(1+x^{2n}\right)$, then the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x=0$ is

Options:

A. 0

B. -1

C. 1

D. 2

Answer: C

Solution:

$y=(1+x)\left(1+x^2\right)\left(1+x^4\right) \ldots\left(1+x^{2n}\right)$ .... (i)

Taking 'log' on both sides, we get

$$\log y=\log(1+x)+\log\left(1+x^2\right)+\log\left(1+x^4\right)+\ldots+\log\left(1+x^{2n}\right)$$

Differentiating w.r.t. x, we get

$\frac{1}{y} \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+x}+\frac{2x}{1+x^2}+\frac{4x^3}{1+x^4}+\ldots+\frac{2\mathrm{n} \times x^{2\mathrm{n}-1}}{1+x^{2\mathrm{n}}}$ ..... (ii)

At $x=0$, (i) $\Rightarrow y=1$

$\therefore$ (ii) $\Rightarrow \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=0}=1+0+0+\ldots+0=1$

Question 44

$\lim\limits_{x \rightarrow 0} \frac{\cos 7x^{\circ}-\cos 2x^{\circ}}{x^2}$ is

Options:

A. $\frac{-45}{2}\pi^2$

B. $\frac{-45}{2}\pi$

C. $\frac{-\pi^2}{1440}$

D. $\frac{-\pi^2}{2880}$

Answer: C

Solution:

$$\lim_{x\to0}\frac{\cos 7x^\circ-\cos 2x^\circ}{x^2}$$
$$=\lim_{x\to0}\frac{\cos\left(\frac{7\pi}{180}\right)x-\cos\left(\frac{2\pi}{180}\right)x}{x^2}$$
$$=\frac{\left(\frac{2\pi}{180}\right)^2-\left(\frac{7\pi}{180}\right)^2}{2}$$
$$\ldots\left[\because\lim_{x\to0}\frac{\cos mx-\cos nx}{x^2}=\frac{n^2-m^2}{2}\right]$$
$$=\frac{-\pi^2}{1440}$$

Question 45

$$\int_0^4|2x-5|dx=$$

Options:

A. $\frac{13}{2}$

B. $\frac{15}{2}$

C. $\frac{17}{4}$

D. $\frac{17}{2}$

Answer: D

Solution:

$$\int_0^4 |2x-5|\mathrm{d}x = \int_0^{\frac{5}{2}} (5-2x)\mathrm{d}x + \int_{\frac{5}{2}}^4 (2x-5)\mathrm{d}x$$

$$= \left[5x - x^2\right]_0^{\frac{5}{2}} + \left[x^2 - 5x\right]_{\frac{5}{2}}^4$$

$$= \frac{25}{4} + \frac{9}{4}$$

$$= \frac{34}{4}$$

$$= \frac{17}{2}$$

Question 46

If λ is the perpendicular distance of a point P on the circle $x^2 + y^2 + 2x + 2y - 3 = 0$, from the line $2x + y + 13 = 0$, then maximum possible value of λ is

Options:

A. $2\sqrt{5}$

B. $3\sqrt{5}$

C. $4\sqrt{5}$

D. $\sqrt{5}$

Answer: B

Solution:

Given equation of the circle is

$x^2 + y^2 + 2x + 2y - 3 = 0$

Which can be written as: $(x+1)^2 + (y+1)^2 = 5$

It is a circle with centre $(-1,-1)$ and radius $\sqrt{5}$

Given line is: $2x+y+13=0$

To find the required distance, we find the equation of a line perpendicular to the given line, and passing through the centre of the given circle.

$\therefore$ Equation of this line is: $(y+1)=\frac{1}{2}(x+1)$ i.e., $x=2y+1$

Now, we find the points where line $x=2y+1$ intersects the circle $x^2+y^2+2x+2y-3=0$

$\therefore\ (2y+1)^2+y^2+2(2y+1)+2y-3=0$
$\therefore\ 4y^2+4y+1+y^2+4y+2+2y-3=0$
$\therefore\ 5y^2+10y=0$
$\therefore\ y(y+2)=0$
$\therefore\ y=0$ or $y=-2$
$\therefore\ x=1$ or $x=-3$

$\therefore$ $(1,0)$ and $(-3,-2)$ are the points on the circle, and one of them is at the maximum distance from the given line.

$\therefore\ d_1=\left|\frac{2(1)+(0)+13}{\sqrt{4+1}}\right|$ and $d_2=\left|\frac{2(-3)+(-2)+13}{\sqrt{4+1}}\right|$
$\therefore\ d_1=\frac{15}{\sqrt{5}}=3\sqrt{5}$ and $d_2=\frac{5}{\sqrt{5}}=\sqrt{5}$
$\therefore\ \lambda=3\sqrt{5}$

Question 47

If the line $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3\mathrm{p}}=\frac{y-5}{1}=\frac{6-\mathrm{z}}{5}$ are at right angles, then $\mathrm{p}=$

Options:

A. $\frac{70}{11}$

B. $\frac{11}{70}$

C. $\frac{-70}{11}$

D. $\frac{-11}{70}$

Answer: A

Solution:

Given lines can be written as

$\frac{x-1}{-3}=\frac{y-2}{\frac{2p}{7}}=\frac{z-3}{2}$ and $\frac{x-1}{-\frac{3p}{7}}=\frac{y-5}{1}=\frac{z-6}{-5}$

As these lines are at right angles, we get

$(-3)\left(-\frac{3p}{7}\right)+\left(\frac{2p}{7}\right)(1)+(2)(-5)=0$

$$\therefore \quad \frac{9p}{7}+\frac{2p}{7}-10=0$$
$$\therefore \quad \frac{11p}{7}=10 \Rightarrow p=\frac{70}{11}$$

Question 48

The value of $\sin\left(\cot^{-1}x\right)$ is

Options:

A. $\frac{1}{\sqrt{1+x^2}}$

B. $\sqrt{1+x^2}$

C. $\frac{1}{x\sqrt{1+x^2}}$

D. $x\sqrt{1+x^2}$

Answer: A

Solution:

$\sin\left(\cot^{-1} x\right)$

Let $\cot^{-1} x = t$

$\therefore \quad x = \cot^{t}$

$\therefore \quad 1 + \cot^2 t = 1 + x^2$

$\therefore \quad \operatorname{cosec}^2 t = 1 + x^2$

$\therefore \quad \operatorname{cosec} t = \sqrt{1 + x^2}$

$\therefore \quad \sin t = \frac{1}{\sqrt{1+x^2}}$

$\therefore \quad t = \sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

$\therefore \quad \sin\left(\cot^{-1} x\right) = \sin\left(\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right)$

$= \frac{1}{\sqrt{1+x^2}}$

Question 49

If $\int \cos^{\frac{3}{5}} x \cdot \sin^3 x dx = \frac{-1}{m} \cos^m x + \frac{1}{n} \cos^n x + c$, (where c is the constant of integration), then $(\mathrm{m}, \mathrm{n}) =$

Options:

A. $\left(\frac{18}{5}, \frac{8}{5}\right)$

B. $\left(\frac{-8}{5}, \frac{18}{5}\right)$

C. $\left(\frac{8}{5}, \frac{18}{5}\right)$

D. $\left(\frac{-18}{5}, \frac{-8}{5}\right)$

Answer: C

Solution:

$$\begin{aligned} \text{Let} \quad \mathrm{I} &= \int \cos^{\frac{3}{5}} x \sin^3 x \, dx \\ &= \int \cos^{\frac{3}{5}} x \left(1 - \cos^2 x\right) \sin x \, dx \\ &= \int \cos^{\frac{3}{5}} x \sin x \, dx - \int \cos^{\frac{13}{5}} x \sin x \, dx \end{aligned}$$

Let $\cos x = \mathrm{t} \Rightarrow -\sin x \, dx = \mathrm{dt}$

$\therefore \quad I=-\int \mathrm{t}^{\frac{3}{5}} \mathrm{dt}+\int \mathrm{t}^{\frac{13}{5}} \mathrm{dt}$

$=\frac{-1}{\left(\frac{8}{5}\right)} \mathrm{t}^{\frac{8}{5}}+\frac{1}{\left(\frac{18}{5}\right)} \mathrm{t}^{\frac{13}{5}}+\mathrm{c}$

$=\frac{-1}{\left(\frac{8}{5}\right)} \cos ^{\frac{8}{5}} x+\frac{1}{\left(\frac{18}{5}\right)} \cos ^{\frac{13}{5}} x+\mathrm{c}$

Comparing with $\frac{-1}{\mathrm{m}} \cos ^{\mathrm{m}} x+\frac{1}{\mathrm{n}} \cos ^{\mathrm{n}} x+\mathrm{c}$, we get $\mathrm{m}=\frac{8}{5}, \mathrm{n}=\frac{18}{5}$

Question 50

Let PQR be a right angled isosceles triangle, right angled at $\mathrm{P}(2,1)$. If the equation of the line QR is $2x+y=3$, then the equation representing the pair of lines PQ and PR is

Options:

A. $3x^2-3y^2+8xy+20x+10y+25=0$

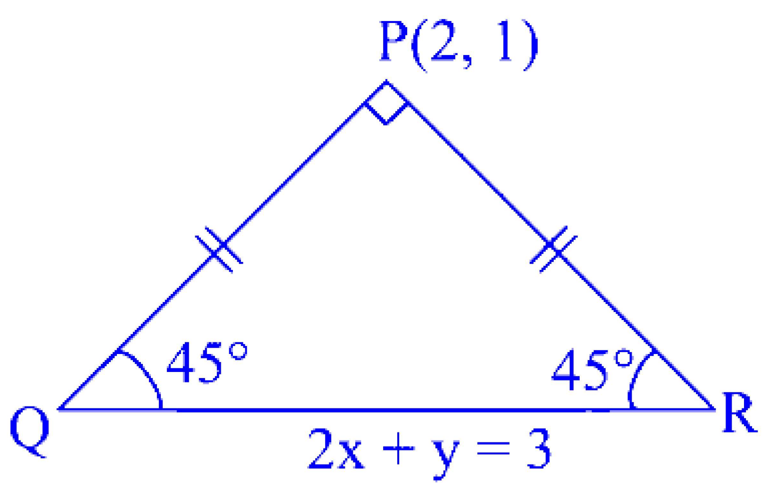
B. $3x^2-3y^2+8xy-20x-10y+25=0$

C. $3x^2-3y^2+8xy+10x+15y+20=0$

D. $3x^2-3y^2-8xy-10x-15y-20=0$

Answer: B

Solution:



Slope of QR $= -2$.

Slope of PQ $= m_1$

$$\therefore \quad \tan 45^\circ = \left|\frac{m_1+2}{1+m_1(-2)}\right|$$

$$\Rightarrow 1 = \left|\frac{m_1+2}{1-2m_1}\right|$$

$$\Rightarrow m_1 = -\frac{1}{3}$$

$\therefore$ Equation of PQ passing through point $P(2,1)$ and having slope $\frac{-1}{3}$ is

$$y-1 = -\frac{1}{3}(x-2)$$

$$\Rightarrow 3(y-1)+(x-2)=0 \quad \text{.... (i)}$$

Slope of PR $= m_2 = 3 \quad \ldots [\because PQ \perp PR]$

$\therefore$ equation of PR is

$$y-1 = 3(x-2)$$
$$\Rightarrow (y-1)-3(x-2)=0 \quad \text{.... (ii)}$$

$\therefore$ The joint equation of the lines is

$$[3(y-1)+(x-2)][(y-1)-3(x-2)]=0$$

$$\Rightarrow 3(y-1)^2 - 8(y-1)(x-2) - 3(x-2)^2 = 0$$

$$\Rightarrow 3\left(x^2-4x+4\right)+8(xy-x-2y+2)$$

$$-3\left(y^2-2y+1\right)=0$$

$$\Rightarrow 3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$$

Question 51

Find $[OH]$ if a monoacidic base is 3% ionised in its 0.04 M solution.

Options:

A. 3.1×10^{-2} mol L^{-1}

B. 4.5×10^{-3} mol L^{-1}

C. 9.0×10^{-2} mol L^{-1}

D. 1.2×10^{-3} mol L^{-1}

Answer: D

Solution:

For a monoacidic base,

$$[OH^-] = c \times \alpha$$
$$[OH^-] = 0.04 \times \frac{3}{100}$$
$$[OH^-] = 1.2 \times 10^{-3} \text{ mol L}^{-1}$$

Question 52

Calculate ΔG° for the reaction $Mg_{(s)} + Sn^{++}_{(aq)} \longrightarrow Mg^{++}_{(aq)} + Sn_{(s)}$ if E^0_{cell} is 2.23 V.

Options:

A. -430.4 kJ

B. 215.2 kJ

C. 645.6 kJ

D. -860.8 kJ

Answer: A

Solution:

$$\begin{aligned}\Delta G^{\circ} &= -nFE^{\circ}_{cell} \\ &= -2 \times 96500 \times 2.23 \\ &= -430390 \text{ J} \\ &= -430.4 \text{ kJ}\end{aligned}$$

Question 53

If lattice enthalpy and hydration enthalpy of KCl are 699 kJ mol^{-1} and -681.8 kJ mol^{-1} respectively. What is the enthalpy of solution of KCl ?

Options:

A. 8.20 kJ mol^{-1}

B. 10.25 kJ mol^{-1}

C. 13.80 kJ mol^{-1}

D. 17.20 kJ mol^{-1}

Answer: D

Solution:

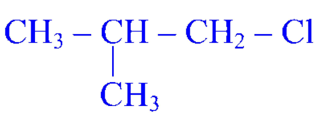
$$\begin{aligned}\Delta_{soln} H &= \Delta_L H + \Delta_{hyd} H \\ &= +699 \text{ kJ mol}^{-1} + \left(-681.8 \text{ kJ mol}^{-1}\right) \\ &= 17.2 \text{ kJ mol}^{-1}\end{aligned}$$

Question 54

Which of the following compounds does NOT undergo Williamson's synthesis?

Options:

A. $C_2H_5 - Cl$

B. 

C. $C_6H_5 - Cl$

D. $CH_3 - CH_2 - CH_2 - Cl$

Answer: C

Solution:

Aryl halides do not give Williamson's synthesis.

Question 55

What is the expression for solubility product of silver chromate if it's solubility is expressed as $\mathrm{S\ mol\ L^{-1}}$?

Options:

A. $2\ S^2$

B. $3\ S^3$

C. $4\ S^3$

D. $27\ S^4$

Answer: C

Solution:

$$\mathrm{Ag_2CrO_{4(s)} \rightleftharpoons 2Ag^+_{(aq)} + CrO^{2-}_{4(aq)}}$$

Here, $x = 2, \mathrm{y} = 1$

$$\therefore \quad \mathrm{K_{sp}} = x^x \mathrm{y^y S^{\mathit{x}+y}} = (2)^2(1)^1\ \mathrm{S}^{2+1} = 4\ \mathrm{S}^3$$

Question 56

Which from following is a non-ferrous alloy?

Options:

A. Nickel steel

B. Chromium steel

C. Stainless steel

D. Brass

Answer: D

Solution:

A non-ferrous alloy is an alloy that does not contain iron in significant amounts. Among the options provided:

- Nickel steel, Chromium steel, and Stainless steel are ferrous alloys because they are primarily composed of iron along with other elements.
- Brass, on the other hand, is an alloy primarily made up of copper and zinc and does not contain iron in significant amounts.

Therefore, the correct answer is:

Option D: Brass

Brass is a non-ferrous alloy.

Question 57

What are the number of octahedral and tetrahedral voids in 0.3 mole substance respectively if it forms hcp structure?

Options:

A. 1.8066×10^{23} and 3.6132×10^{23}

B. 3.6132×10^{23} and 1.8066×10^{23}

C. 6.022×10^{23} and 12.044×10^{23}

D. 12.044×10^{23} and 6.022×10^{23}

Answer: A

Solution:

Number of atoms in 0.3 mol

$$
\begin{aligned}
& = 0.3 \times N_A \\
& = 0.3 \times 6.022 \times 10^{23} \\
& = 1.8066 \times 10^{23}
\end{aligned}
$$

i. For hcp structure, Number of octahedral voids

$=$ Number of atoms

$= 1.8066 \times 10^{23}$

ii. For hcp structure, Number of tetrahedral voids

$= 2\times$ Number of atoms

$$
\begin{aligned}
& = 2 \times 1.8066 \times 10^{23} \\
& = 3.6132 \times 10^{23}
\end{aligned}
$$

Question 58

Calculate the molar mass of an element having density 7.8 g cm^{-3} that forms bcc unit cell. $\left[a^3 \cdot N_A = 16.2 \text{ cm}^3 \text{ mol}^{-1}\right]$

Options:

A. 63.18 g mol^{-1}

B. 61.23 g mol^{-1}

C. 59.31 g mol^{-1}

D. 65.61 g mol^{-1}

Answer: A

Solution:

For bcc unit cell, $n = 2$.

$$\text{Density } (\rho) = \frac{M\,\text{n}}{a^3 N_A}$$

$$7.8 = \frac{M \times 2}{16.2}$$

$$M = \frac{7.8 \times 16.2}{2} = 63.18 \text{ g mol}^{-1}$$

Question 59

Which among the following compounds exhibits +2 oxidation state of oxygen?

Options:

A. H_2O

B. SO_2

C. OF_2

D. H_2O_2

Answer: C

Solution:

The oxidation number of F is -1 in all of its compounds. Hence, in OF_2 oxidation number of oxygen is +2.

Question 60

Identify substrate A in the following reaction.

$$\text{A} + \text{AgOH} \xrightarrow[\text{ii) } \Delta]{\text{i) moist} \quad \text{Ag}_2\text{O}} \text{CH}_3\text{CH}_2\,\text{N}(\text{CH}_3)_2 + \text{CH}_2 = \text{CH}_2$$

Options:

A. Diethyldimethyl ammonium halide

B. Ethyltrimethyl ammonium halide

C. Diethyldimethyl ammonium hydroxide

D. Ethyltrimethyl ammonium hydroxide

Answer: A

Solution:

The reaction is Hofmann elimination and substrate 'A' is diethyldimethyl ammonium halide.

$$(CH_3CH_2)_2\overset{+}{N}(CH_3)_2X^- + AgOH \xrightarrow[ii)\ \Delta]{i)\ moist\ Ag_2O} CH_3CH_2N(CH_3)_2 + CH_2 = CH_2$$

Question 61

What volume of $CO_{2(g)}$ at STP is obtained by complete combustion of 6 g carbon?

Options:

A. 22.4 dm^3

B. 11.2 dm^3

C. 5.6 dm^3

D. 2.24 dm^3

Answer: B

Solution:

$$C_{(s)} + O_{2(g)} \longrightarrow CO_{2(g)}$$
$$1 \text{ mol C} \equiv 1 \text{ mol } CO_{2(g)}$$
$$6 \text{ gC} = 0.5 \text{ mol C}$$
$$\therefore \quad 0.5 \text{ molC} \equiv 0.5 \text{ mol } CO_{2(g)}$$
$$\text{At STP, } 1 \text{ mol } CO_{2(g)} \equiv 22.4 \text{ dm}^3$$
$$\therefore \quad 0.5 \text{ mol } CO_{2(g)} = 11.2 \text{ dm}^3$$

Question 62

Identify the chiral molecule from the following.

Options:

A. 2-Iodopropane

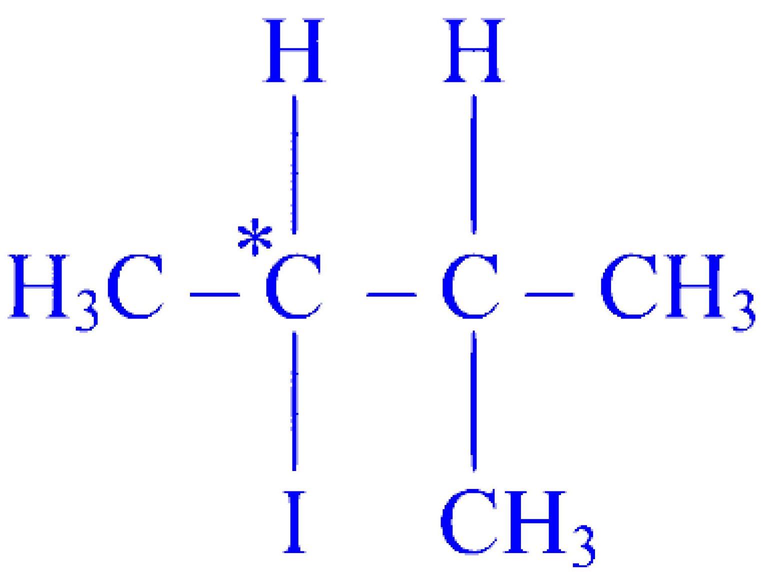
B. 2-Iodo-2-methylbutane

C. 2-lodo-3-methylbutane

D. 3-Iodopentane

Answer: C

Solution:



2-Iodo-3-methylbutane

Question 63

Calculate the time needed for reactant to decompose 99.9% if rate constant of first order reaction is 0.576 minute^{-1}.

Options:

A. 8 minutes

B. 12 minutes

C. 16 minutes

D. 20 minutes

Answer: B

Solution:

99.9% of the reaction is complete.

So, if $[A]_0 = 100$, then $[A]_t = 100 - 99.9 = 0.1$

$$\begin{aligned} t &= \frac{2.303}{k} \log_{10} \frac{[A]_0}{[A]_t} \\ &= \frac{2.303}{0.576} \log_{10} \frac{100}{0.1} = \frac{2.303}{0.576} \log_{10}(1000) \\ &= \frac{2.303}{0.576} \times 3 \\ &= 11.99 \approx 12 \text{ minutes} \end{aligned}$$

Question 64

What is the number of moles of sp^3 hybrid carbon atoms in one mole of 2-Methylbut-2-ene?

Options:

A. Four

B. Three

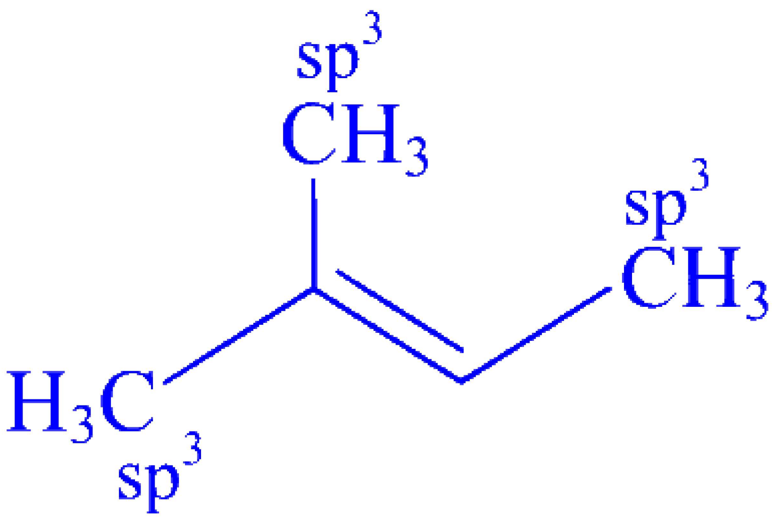
C. Two

D. One

Answer: B

Solution:

Structure of 2-Methylbut-2-ene:



In 2-methylbut-2-ene, three carbon atoms are sp^3 hybridized.

Therefore, in one mole of 2-methylbut-2-ene, there are three moles of sp^3 hybrid carbon atoms.

Question 65

Identify major product A in following reaction.

3-Bromo-2-methylpentane $\xrightarrow[\Delta]{\text{alc.KOH}}$ **A**

Options:

A. 2-Methylpentan-3-ol

B. 2-Methylpent-2-ene

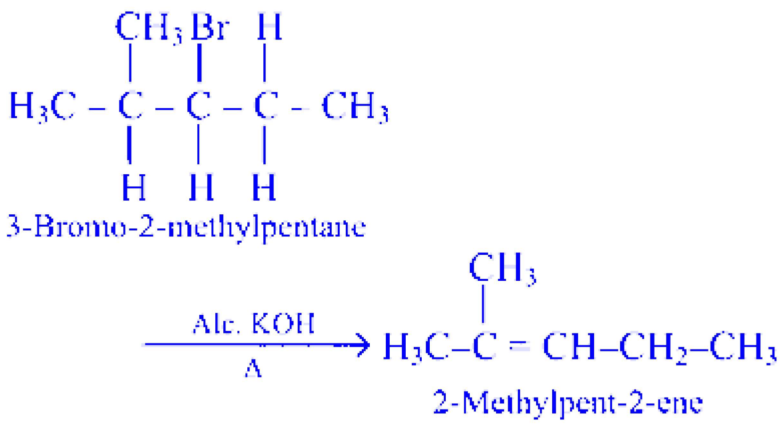
C. 4-Methylpent-3-ene

D. 4-Methylpentan-3-ol

Answer: B

Solution:

When alkyl halide having at least one β-hydrogen is boiled with alcoholic solution of potassium hydroxide, it undergoes elimination of hydrogen atom from β-carbon and halogen atom from α-carbon resulting in the formation of an alkene. This reaction is known as dehydrohalogenation reaction or β-elimination. The preferred product is that alkene which has greater number of alkyl groups attached to doubly bonded carbon atoms according to Saytzeff rule.



Question 66

For reaction, $CO_{(g)} + \frac{1}{2}O_{2(g)} \longrightarrow CO_{2(g)}$

Which of the following equations is CORRECT at constant T **and** P **?**

Options:

A. $\Delta H < \Delta U$

B. $\Delta H > \Delta U$

C. $\Delta H = \Delta U$

D. $\Delta H = 0$

Answer: A

Solution:

$$\Delta H = \Delta U + \Delta n_g RT$$
$$\Delta n_g = 1 - (1 + 0.5) = -0.5$$
$$\Delta H = \Delta U - 0.5$$
$$\Rightarrow \Delta H < \Delta U$$

$\Delta n_g = -ve \Rightarrow \Delta H < \Delta U$

Therefore, option (A) is correct.

Question 67

Identify the example of zero-dimensional nanostructure from following.

Options:

A. Nanotubes

B. Fibres

C. Thin films

D. Quantum dots

Answer: D

Question 68

What is pH of solution containing 50 mL each of 0.1 M sodium acetate and 0.01 M acetic acid? $(pK_a CH_3COOH = 4.50)$

Options:

A. 2.5

B. 3.5

C. 4.5

D. 5.5

Answer: D

Solution:

For acidic buffer solution,

$$\mathrm{pH}=\mathrm{pK_a}+\log_{10}\frac{[\text{ Salt }]}{[\text{ Acid }]};$$
$$\mathrm{pH}=4.50+\log_{10}\frac{0.1}{0.01};$$
$$\mathrm{pH}=4.50+\log 10; \mathrm{pH}=5.50$$

For acidic buffer, if [Salt] $>$ [Acid], then pH $>$ $\mathrm{pK_a}$ of acid. Hence, only option (D) is valid.

Question 69

Calculate amount of methane formed by liberation of $149.6 \mathrm{~kJ}$ of heat using following equation.

$$\mathrm{C}_{(\mathrm{s})}+2 \mathrm{H}_{2(\mathrm{~g})} \longrightarrow \mathrm{CH}_{4(\mathrm{~g})} \quad \Delta \mathrm{H}=-74.8 \mathrm{~kJ} / \mathrm{mol}$$

Options:

A. 16 g

B. 24 g

C. 32 g

D. 48 g

Answer: C

Solution:

According to the given reaction, 74.8 kJ of heat is evolved when 1 mol of methane is formed.

$74.8 \text{ kJ} \equiv 1 \text{ mol } CH_4 = 16 \text{ g } CH_4$
$149.6 \text{ kJ} \equiv x \text{ g } CH_4$
$$x = \frac{149.6 \times 16}{74.8} = 32 \text{ g}$$

Question 70

Which from following polymers is used to obtain tyre cords?

Options:

A. Nylon 6

B. Polyacrylonitrile

C. Bakelite

D. Terylene

Answer: A

Solution:

Nylon 6 is the polymer commonly used in the production of tire cords. Tire cords are a crucial component of tire manufacturing, providing the necessary strength and durability. Nylon 6, known for its high tensile strength, abrasion resistance, and durability, is well-suited for this application.

Therefore, the correct answer is :

Option A : Nylon 6

Question 71

Electrolytic cells containing Zn and Al salt solutions are connected in series. If 6.5 g of Zn is deposited in one cell calculate mass of Al deposited in second cell (molar mass : $\mathrm{Zn}=65, \mathrm{Al}=27$) by passing definite quantity of electricity?

Options:

A. 2.4 g

B. 2.1 g

C. 2.7 g

D. 1.8 g

Answer: D

Solution:

Cell 1:

$$Zn^{2+}_{(sq)} + 2e^- \longrightarrow Zn_{(s)}$$

$$(\text{mole ratio})_1 = \frac{1 \text{ mol}}{2 \text{ mole}^-}$$

Cell 2:

$$Al^{3+}_{(aq)} + 3e^- \longrightarrow Al_{(s)}$$

$$(\text{ mole ratio })_2 = \frac{1 \text{ mol}}{3 \text{ mole}^-}$$

$$\frac{W_1}{(\text{ mole ratio })_1 \times M_1} = \frac{W_2}{(\text{ mole ratio })_2 \times M_2}$$

$$\frac{6.5 \text{ g}}{1 \text{ mol}/2 \text{ mole}^- \times 65 \text{ g mol}^{-1}} = \frac{W_2}{1 \text{ mol}/3 \text{ mole}^- \times 27 \text{ g mol}^{-1}}$$

$$\frac{6.5 \text{ g} \times 2}{65} = \frac{W_2 \times 3}{27}$$

$$W_2 = \frac{6.5 \times 2 \times 27}{65 \times 3} = 1.8 \text{ g}$$

Question 72

What type of glycosidic linkages are present in cellulose?

Options:

A. $\beta - 1, 6$

B. $\beta-1,4$

C. $\alpha-1,6$

D. $\alpha-1,4$

Answer: B

Solution:

Cellulose is a straight chain polysaccharide of β-glucose units linked by $\beta-1,4$-glycosidic bonds.

Question 73

Calculate the rate constant of first order reaction if half life of reaction is 40 minutes.

Options:

A. 1.733×10^{-2} minute $^{-1}$

B. 1.951×10^{-2} minute $^{-1}$

C. 1.423×10^{-2} minute $^{-1}$

D. 1.256×10^{-2} minute $^{-1}$

Answer: A

Solution:

For a first order reaction, $k=\frac{0.693}{t_{1/2}}$

$\mathrm{k}=\frac{0.693}{40}=1.733 \times 10^{-2}$ minute $^{-1}$

Question 74

Identify product 'B' in following sequence of reactions.

$$2n \text{ Propanone} \xrightarrow{Ba(OH)_2} A \xrightarrow[-H_2O]{\Delta} nB$$

Options:

A. 4-Hydroxy-4-methylpentan-2-one

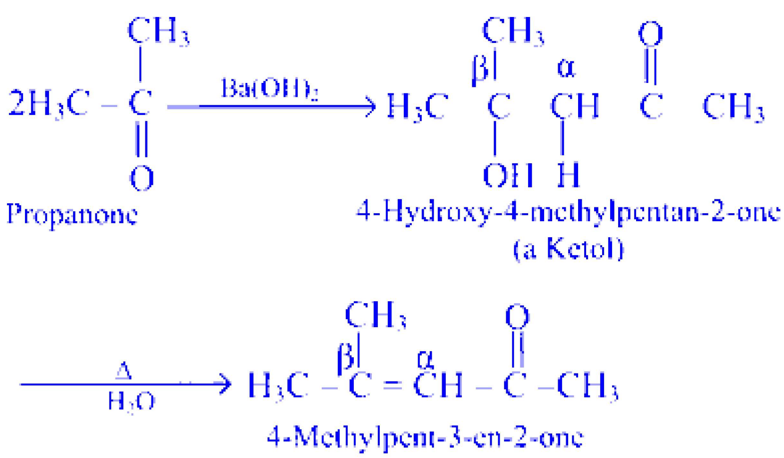
B. 2-Methylpentan-3-one

C. 2-Methylpent-2-en-4-one

D. 4-Methylpent-3-en-2-one

Answer: D

Solution:



Question 75

Identify rate law expression for $2NO_{(g)} + Cl_{2(g)} \rightarrow 2NOCl_{(g)}$ if the reaction is second order in NO and first order in Cl_2.

Options:

A. Rate $= k[NO]^2 [Cl_2]$

B. Rate $= k[NO] [Cl_2]$

C. Rate $= k[NO]^2$

D. Rate $= k[Cl_2]$

Answer: A

Question 76

Which among the following solutions has minimum boiling point elevation?

Options:

A. 0.1 m NaCl

B. 0.2 m KNO_3

C. 0.1 m Na_2SO_4

D. 0.05 m $CaCl_2$

Answer: D

Solution:

$NaCl \longrightarrow Na^+ + Cl^-$

Total ions $= 0.1 + 0.1 = 0.2$ ions

$KNO_3 \longrightarrow K^+ + NO_3^-$

Total ions $= 0.2 + 0.2 = 0.4$ ions

$Na_2SO_4 \longrightarrow 2Na^+ + SO_4^{2-}$

Total ions $= 0.2 + 0.1 = 0.3$ ions

$CaCl_2 \longrightarrow Ca^{2+} + 2Cl^-$

Total ions $= 0.05 + 0.1 = 0.15$ ions

0.05 m$CaCl_2$ solution has minimum ions in solution, so it shows minimum boiling point elevation.

Question 77

Calculate osmotic pressure of solution of 0.025 mole glucose in 100 mL **water at** 300 K. $\left[\mathrm{R}=0.082\ \mathrm{atmdmmol}^{-1}\ \mathrm{K}^{-1}\right]$

Options:

A. 1.54 atm

B. 2.05 atm

C. 6.15 atm

D. 3.08 atm

Answer: C

Solution:

$$\begin{aligned}\pi &= \mathrm{MRT} = \frac{\mathrm{n_2RT}}{\mathrm{V}} \\ &= \frac{0.025\ \mathrm{mol} \times 0.082\ \mathrm{dm}^3\ \mathrm{atm\ mol}^{-1}\ \mathrm{K}^{-1} \times 300\ \mathrm{K}}{0.1\mathrm{dm}^3} \\ &= 6.15\ \mathrm{atm}\end{aligned}$$

Question 78

Which from following is a neutral ligand?

Options:

A. Aqua

B. Sulphato

C. Carbonato

D. Bromo

Answer: A

Question 79

How many isomers of C_4H_{11} N are tertiary amines?

Options:

A. One

B. Two

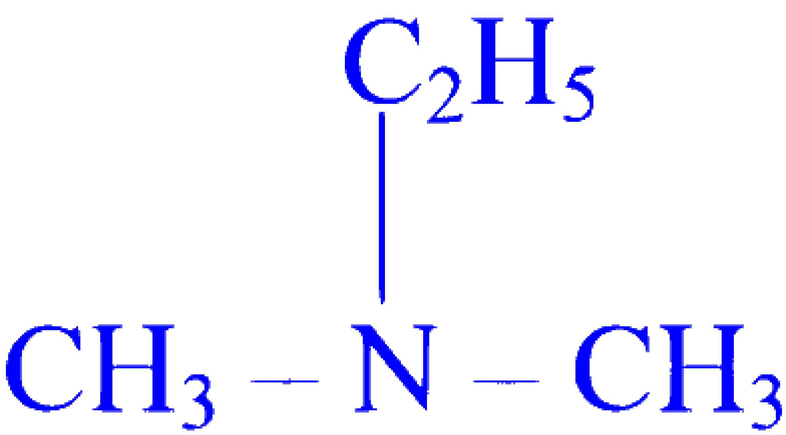
C. Three

D. Four

Answer: A

Solution:

Only one isomer of $C_4H_{11}N$ is a tertiary amine.



Question 80

Which element from following exhibits diagonal relationship with Mg ?

Options:

A. Be

B. Li

C. Na

D. B

Answer: B

Question 81

Identify the good conductor of electricity from following band gap energy values of solids.

Solid	E gap
A	5.47 eV
B	0.0 eV
C	1.12 eV
D	0.67 eV

Options:

A. A

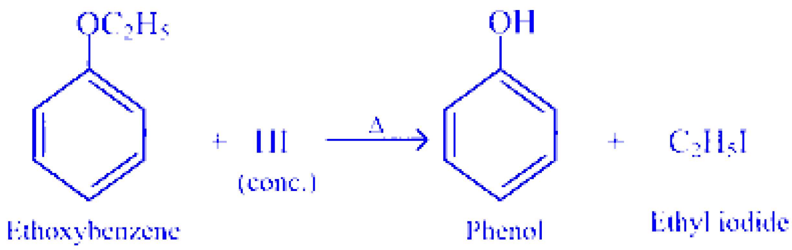
B. B

C. C

D. D

Answer: B

Solution:



According to band theory, solids with smaller band gap energies are good conductors of electricity. Therefore, based on the given band gap energy values, the good conductor of electricity would be solid B.

Question 82

Identify the product obtained when ethoxybenzene reacts with hot and concentrated HI.

Options:

A. Ethyl iodide and Phenol

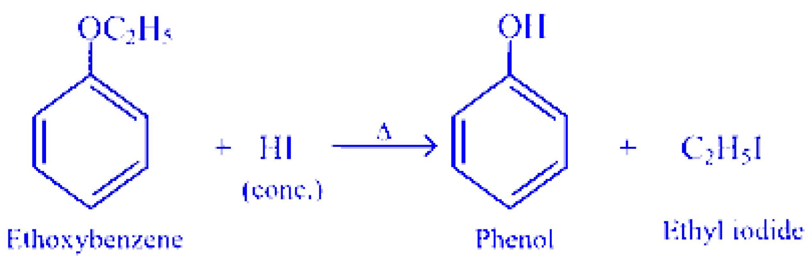
B. Ethyl alcohol and Phenol

C. Ethyl alcohol and Iodobenzene

D. Ethyl iodide and Iodobenzene

Answer: A

Solution:



Aryl alkyl ethers have stronger and shorter bond between oxygen and the aromatic ring. Hence, they undergo cleavage of oxygen - alkyl bond and yield phenol and alkyl halide on reaction with HI.

Question 83

Identify thermosetting polymer from following

Options:

A. Urea formaldehyde resin

B. Polythene

C. Polystyrene

D. Polyvinyls

Answer: A

Solution:

Thermosetting polymers are a type of polymer that become irreversibly hardened upon being cured. Among the options provided, the thermosetting polymer is :

Option A : Urea formaldehyde resin

Urea formaldehyde resin is a well-known example of a thermosetting polymer. Once it is set through a curing process, it cannot be melted and reshaped, which is a characteristic property of thermosetting polymers.

Polythene, Polystyrene, and Polyvinyls (Polyvinyl Chloride or PVC), on the other hand, are examples of thermoplastic polymers, which can be melted and reshaped upon heating.

Question 84

Which from following phenomena is inversely proportional with adsorption?

Options:

A. Critical temperature of gas

B. Surface area of adsorbent

C. Temperature of process

D. Pressure of gas

Answer: C

Solution:

Adsorption is the process where molecules or atoms adhere to a surface. The extent of adsorption depends on various factors, but when considering its relationship with other phenomena, we look at how changes in these factors influence the amount of adsorption.

- **Critical Temperature of Gas** : The critical temperature of a gas is the temperature above which it cannot be liquefied, regardless of the pressure applied. Generally, gases which can be easily liquefied (i.e., gases with higher critical temperatures) are adsorbed to a greater extent because they are easier to condense on surfaces. However, this isn't an inverse relationship.

- **Surface Area of Adsorbent** : The greater the surface area of the adsorbent, the more sites are available for adsorption, leading to increased adsorption. This is a direct, not inverse, relationship.

- **Temperature of Process** : Generally, adsorption is exothermic (releases heat). According to Le Chatelier's principle, increasing the temperature of an exothermic process will decrease the extent of the reaction. Therefore, as temperature increases, adsorption typically decreases, indicating an inverse relationship.

- **Pressure of Gas** : For gases, increasing pressure generally increases adsorption because more gas molecules are forced into proximity with the adsorbent surface. This is a direct relationship.

Based on these considerations, the correct option is :

Option C : Temperature of process (as it has an inverse relationship with adsorption).

Question 85

Calculate the frequency of blue light having wavelength 440 nm.

Options:

A. $6.82 \times 10^{14} \text{ Hz}$

B. $7.5 \times 10^{14} \text{ Hz}$

C. $4.0 \times 10^{14} \text{ Hz}$

D. $5.26 \times 10^{14} \text{ Hz}$

Answer: A

Solution:

$\nu=\frac{c}{\lambda}=\frac{3.0\times10^{8}\ \mathrm{m\ s}^{-1}}{440\times10^{-9}\ \mathrm{m}}=6.82\times10^{14}\ \mathrm{Hz}$

Question 86

Which from following elements is NOT radioactive?

Options:

A. At

B. Po

C. Rn

D. Ar

Answer: D

Question 87

Which from following is strongest reducing agent?

Options:

A. K

B. Al

C. Mg

D. Ag

Answer: A

Solution:

The strength of reducing agents increases from top to bottom in the electrochemical series as E^0 values decrease. Among the given options, K has the lowest value of $\mathrm{E}^{\circ}(-2.925 \mathrm{~V})$.

Question 88

What is the numerical value of spin only magnetic moment of copper in +2 state?

Options:

A. 0.0

B. 1.73

C. 2.78

D. 4.4

Answer: B

Solution:

$\mathrm{Cu}(\mathrm{Z}=29):[\mathrm{Ar}] 3 \mathrm{~d}^{10} 4 \mathrm{~s}^1$
$\mathrm{Cu}^{2+}:[\mathrm{Ar}] 3 \mathrm{~d}^9$
No. of unpaired $\mathrm{e}^{-}=1$
$\mu=\sqrt{\mathrm{n}(\mathrm{n}+2)}=\sqrt{1(1+2)}=\sqrt{3}=1.73 \mathrm{BM}$

Since the number of unpaired electrons $(\mathrm{n})=1, \mu \approx 2$. Hence, option (B) is the correct answer.

Question 89

Identify the element having highest density from following.

Options:

A. O

B. S

C. Se

D. Te

Answer: D

Solution:

In group 16, density increases down the group.

Question 90

What is the shape of AB_4E type of molecule according to VSEPR?

Options:

A. See saw

B. Bent

C. Trigonal pyramidal

D. T shape

Answer: A

Question 91

The molecular formula of hexachlorobenzene is

Options:

A. $C_6H_6Cl_6$

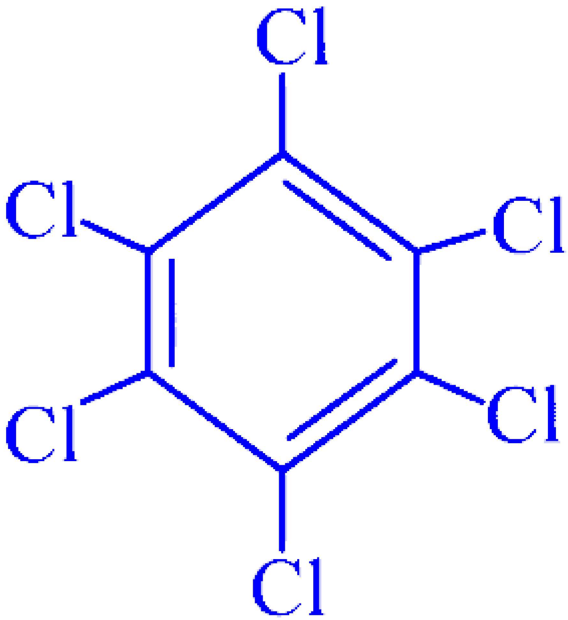
B. C_6Cl_6

C. C_6H_5Cl

D. C_6H_6Cl

Answer: B

Solution:



Hexachlorobenzene (C_6Cl_6)

Question 92

What is the value of specific rotation exhibited by fructose molecule?

Options:

A. $+52.7^\circ$

B. -92.4°

C. $+66.5^\circ$

D. -40.3°

Answer: B

Question 93

Which of the following reactions is Rosenmund reduction?

Options:

A.

$$R-\underset{\underset{O}{||}}{C}-Cl \xrightarrow[\text{Pd-BaSO}_4]{H_2} R-CHO + HCl$$

B.

$$R-CN \xrightarrow[H_2O]{SnCl_2, HCl} R-CHO + NH_4Cl$$

C.

$$R-CHO \xrightarrow[\Delta]{\text{Zn-Hg, Conc.HCl}} RCH_3 + H_2O$$

D.

$$R-\underset{\underset{O}{||}}{C}-R \xrightarrow[\text{ii) KOH, HO}-CH_2-CH_2-OH]{\text{i) } H_2N-NH_2,\ \Delta} R-CH_2-R$$

Answer: A

Question 94

Which from following complexes contains only anionic ligands?

Options:

A. Tetraamminedibromoplatinum (IV) bromide

B. Potassium trioxalatoaluminate (III)

C. Pentaaquaisothiocyanatoiron (III) ion

D. Pentaammineaquacobalt (III) iodide

Answer: B

Solution:

Potassium trioxalatoaluminate (III): $K_3[Al(C_2O_4)_3]$ Oxalate ion $\left(C_2O_4^{2-}\right)$ is an anionic ligand.

Question 95

A hot air balloon has volume of $2000\ dm^3$ at $99^\circ C$. What is the new volume if air in balloon cools to $80^\circ C$?

Options:

A. $2428.9\ dm^3$

B. $2656.9\ dm^3$

C. $2814.9\ dm^3$

D. $1897.8\ dm^3$

Answer: D

Solution:

Using Charles' law,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\therefore \quad \frac{2000 dm^3}{372\ K} = \frac{V_2}{353\ K}$$

$$\therefore \quad V_2 = \frac{2000 \times 353}{372} = 1897.8\ dm^3$$

According to Charles' law, at constant pressure, the volume of a fixed mass of a gas is directly proportional to its temperature in kelvin. Therefore, as temperature decreases, volume will decrease. Hence, only option (D) is valid.

Question 96

Identify the product obtained in following reaction.

n $CH_3MgI + H_2O \xrightarrow{\text{dry ether}}$ **product**

Options:

A. n MgI and n CH_4

B. $\frac{n}{2}C_2H_6$

C. n CH_3OH and n MgI

D. n CH_4 and n MgI(OH)

Answer: D

Solution:

This is the reaction of methylmagnesium iodide (CH_3MgI), a Grignard reagent, with water (H_2O). The general reaction of a Grignard reagent with water is:

$R - MgX + H_2O \rightarrow R - H + MgX(OH)$

where R-MgX is the Grignard reagent (in this case, CH_3MgI, where R = CH_3 and X = I) and R-H is the corresponding hydrocarbon (in this case, CH_4).

So, when CH_3MgI reacts with water, the product will be methane (CH_4) and magnesium hydroxide iodide (MgI(OH)).

Therefore, the correct option is :

Option D : n CH_4 and n MgI(OH).

Question 97

Which of following pairs is an example of isoelectronic species?

Options:

A. $O^{--}; Na^{+}$

B. $O^{--}; F$

C. $K; Ca^{++}$

D. $Ar; Al^{+3}$

Answer: A

Solution:

Atoms and ions having the same number of electrons are isoelectronic species. O^{--} and Na^{+} containing 10 electrons each are isoelectronic species.

Question 98

Which from following compounds is obtained when anisole is heated with dilute sulfuric acid?

Options:

A. Phenol and ethanol

B. Phenol and methanol

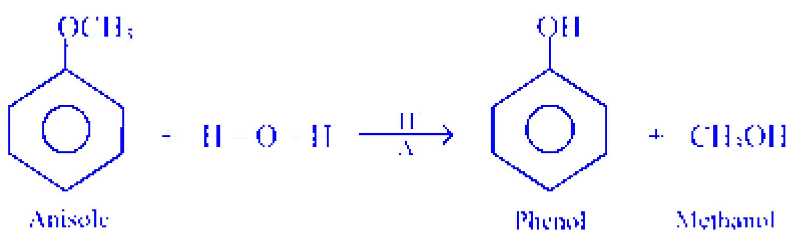
C. Pyrogallol and methanol

D. Phloroglucinol and ethanol

Answer: B

Solution:

Ethers when heated with dilute sulfuric acid under pressure undergo hydrolysis to give alcohols/phenols.



Question 99

Calculate molality of solution of a nonvolatile solute having boiling point elevation 1.89 K if boiling point elevation constant of solvent is $3.15 \text{ K kg mol}^{-1}$.

Options:

A. 0.4 m

B. 0.8 m

C. 0.6 m

D. 0.3 m

Answer: C

Solution:

$$\Delta T_b = K_b \times m$$
$$1.89 = 3.15 \times m$$
$$\therefore \quad m = \frac{1.89}{3.15} = 0.6 \text{ mol kg}^{-1}$$

Question 100

What type of following phenomena does the Cannizzaro reaction exhibit?

Options:

A. Nucleophilic addition

B. Elimination

C. Disproportionation

D. Decomposition

Answer: C

Physics

Question 101

A uniform string is vibrating with a fundamental frequency 'n'. If radius and length of string both are doubled keeping tension constant then the new frequency of vibration is

Options:

A. 2n

B. 3n

C. $\frac{n}{4}$

D. $\frac{n}{3}$

Answer: C

Solution:

$$l_2 = 2l_1, R_2 = 2R_1,\ T_1 = T_2$$

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$\text{Where, m} = \text{ mass per unit length } = \frac{(\pi R^2 l)\rho}{l}$$

$$\therefore \quad m \propto R^2$$

$$\therefore \quad \frac{n_2}{n_1} = \frac{l_1}{l_2} \times \frac{R_1}{R_2}$$

$$\therefore \quad \frac{n_2}{n_1} = \frac{l_1}{2l_1} \times \frac{R_1}{2R_1}$$

$$\therefore \quad n_2 = \frac{n_1}{4} = \frac{n}{4}$$

Question 102

Let γ_1 be the ratio of molar specific heat at constant pressure and molar specific heat at constant volume of a monoatomic gas and γ_2 be the similar ratio of diatomic gas. Considering the diatomic gas molecule as a rigid rotator, the ratio $\frac{\gamma_2}{\gamma_1}$ is

Options:

A. $\frac{37}{21}$

B. $\frac{27}{35}$

C. $\frac{21}{25}$

D. $\frac{35}{27}$

Answer: C

Solution:

For monoatomic gas,

$\gamma_1 = \frac{5}{3}$

For rigid diatomic gas,

$$\gamma_2 = \frac{7}{5}$$

$$\therefore \quad \frac{\gamma_2}{\gamma_1} = \frac{7}{5} \times \frac{3}{5} = \frac{21}{25}$$

Question 103

A railway track is banked for a speed ',v' by elevating outer rail by a height 'h' above the inner rail. The distance between two rails is 'd' then the radius of curvature of track is ($\mathrm{g}=$ gravitational acceleration)

Options:

A. $\frac{\mathrm{v}^2 \mathrm{~d}}{\mathrm{gh}}$

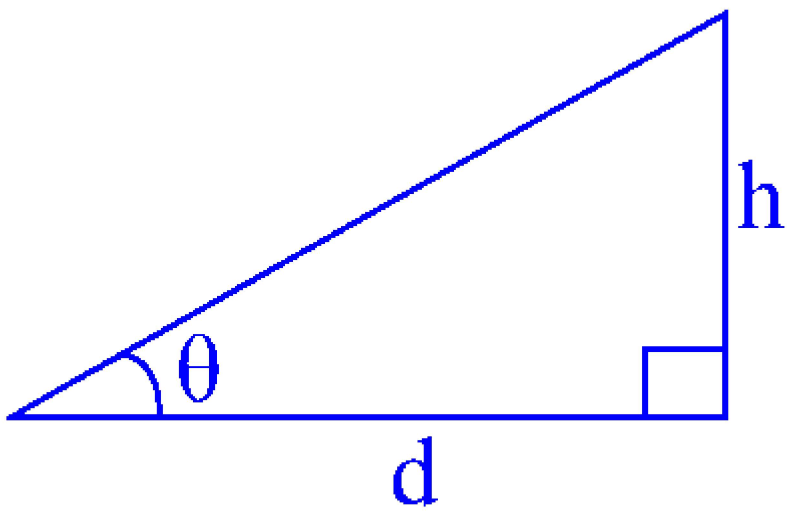
B. $\frac{2v^2}{gdh}$

C. $\frac{gd}{2v^2h}$

D. $\frac{v^2}{2ghd}$

Answer: A

Solution:



From figure,

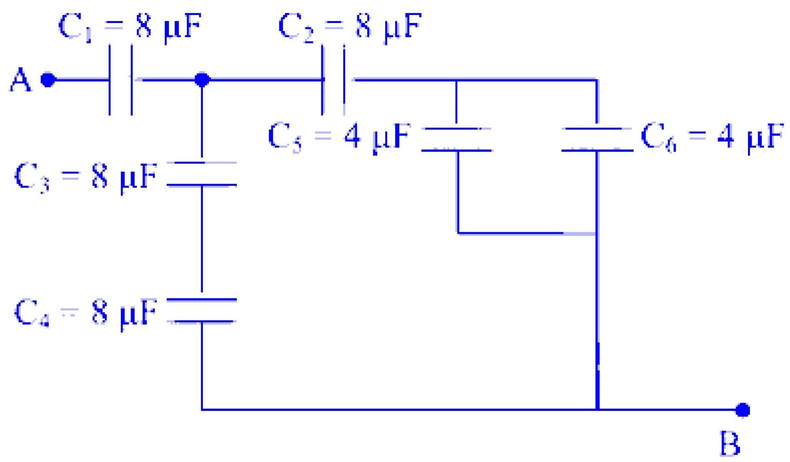
$$\tan\theta = \frac{h}{d}$$

$$\therefore \quad \frac{v^2}{rg} = \frac{h}{d} \quad \ldots.\left(\because \tan\theta = \frac{v^2}{rg}\right)$$

$$\therefore \quad r = \frac{v^2\,d}{gh}$$

Question 104

In the given capacitive network the resultant capacitance between point A and B is



Options:

A. 8 μF

B. 4 μF

C. 2 μF

D. 16 μF

Answer: B

Solution:

In the given circuit, C_3 and C_4 are in series and $C_3 = C_4 = 8\mu F$

$\therefore \quad \frac{1}{C_S} = \frac{1}{C_3} + \frac{1}{C_4}$

$$\therefore \quad C_S = \frac{C_3^2}{2C_3}$$

$$C_S = \frac{C_3}{2}$$

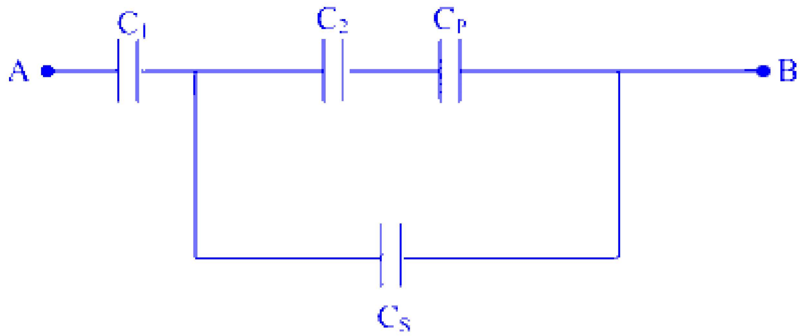
$$\therefore \quad C_S = 4\mu F \quad \text{.... (i)}$$

C_5 and C_6 are in parallel and $C_5 = C_6 = 4\ \mu F$

$C_P = C_5 + C_6$

$\therefore \quad C_P = 8\mu F$

$\therefore$ Equivalent circuit is as shown in figure.



Now, C_2 and C_P are in series and their combination in parallel with C_S

$$\therefore \quad C_E = \frac{C_2 C_p}{C_2 C_p} + C_s$$

$$C_E = \frac{(8)(8)}{16} + 4$$

$$\therefore \quad C_E = 8\mu F$$

Now, C_1 and C_E are in series,

$$\therefore \quad C = \frac{C_1 C_E}{C_1 + C_E}$$

$$\therefore \quad C = \frac{(8)(8)}{16}$$

$$\therefore \quad C = 4\mu F$$

Question 105

In Young's double slit experiment the intensities at two points, for the path difference $\frac{\lambda}{4}$ and $\frac{\lambda}{3}$ (λ = wavelength of light used) are I_1 and I_2 respectively. If I_0 denotes the intensity produced by each one of the individual slits then $\frac{I_1+I_2}{I_0}$ is equal to $\left(\cos 60^\circ = 0.5, \cos 45^\circ = \frac{1}{\sqrt{2}}\right)$

Options:

A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

Phase difference, $\phi = \frac{2\pi}{\lambda} \Delta l$

For first point, $\phi_1 = \frac{2\pi}{\lambda}\left(\frac{\lambda}{4}\right)$

$$\therefore \quad \phi_1 = \frac{\pi^c}{2}$$
$$I_1 = 2I_0\left(1+\cos\phi_1\right)$$
$$\therefore \quad I_1 = 2I_0 \quad \ldots[\because \cos\phi_1 = \cos(\pi/2) = 0]$$

Similarly, for second point, $\phi_2 = \frac{2\pi^c}{3}$

$$\therefore \quad I_2 = 2I_0\left(1+\cos\phi_2\right)$$
$$\therefore \quad I_2 = 2I_0\left(1-\frac{1}{2}\right) \quad \ldots\left[\because \cos\left(\frac{2\pi}{3}\right) = \frac{-1}{2}\right]$$
$$\therefore \quad I_2 = I_0$$

Hence, $\frac{I_1+I_2}{I_0} = \frac{2I_0+I_0}{I_0} = 3$

Question 106

A simple pendulum performs simple harmonic motion about $x = 0$ with an amplitude 'a' and time period 'T'. The speed of the pendulum at $x = \frac{a}{2}$ is

Options:

A. $\frac{\pi a}{T}$

B. $\frac{3\pi^2 a}{T}$

C. $\frac{\pi a\sqrt{3}}{T}$

D. $\frac{\pi a\sqrt{3}}{2}$

Answer: C

Solution:

$$v = \omega\sqrt{a^2 - x^2}$$

At $x = \frac{a}{2}$,

$$\therefore \quad v = \omega\sqrt{a^2 - \frac{a^2}{4}}$$

$$= \omega\frac{\sqrt{3}a}{2}$$

$$= \frac{2\pi}{T} \times \frac{\sqrt{3}a}{2} \quad \ldots\ldots \left(\because \omega = \frac{2\pi}{T}\right)$$

$$\therefore \quad v = \frac{\pi a\sqrt{3}}{T}$$

Question 107

The molar specific heat of an ideal gas at constant pressure and constant volume is C_p and C_v respectively. If R is universal gas constant and $\gamma = \frac{C_p}{C_v}$ then $C_v =$

Options:

A. $\frac{1-\gamma}{1+\gamma}$

B. $\frac{1+\gamma}{1-\gamma}$

C. $\frac{\gamma-1}{R}$

D. $\frac{R}{\gamma-1}$

Answer: D

Solution:

$C_p - C_v = R$

Dividing both the sides by C_v,

$\therefore \quad \gamma - 1 = \frac{R}{C_v} \quad \ldots \left(\because \frac{C_p}{C_v} = \gamma\right)$

$\therefore \quad C_v = \frac{R}{\gamma-1}$

Question 108

Resistance of a potentiometer wire is $2\Omega/\mathrm{m}$. A cell of e.m.f. $1.5\ \mathrm{V}$ balances at $300\ \mathrm{cm}$. The current through the wire is

Options:

A. $2.5\ \mathrm{mA}$

B. $7.5\ \mathrm{mA}$

C. $250\ \mathrm{mA}$

D. $750\ \mathrm{mA}$

Answer: C

Solution:

$l = 300\ \mathrm{cm} = 3\ \mathrm{m}$

Total resistance of wire,

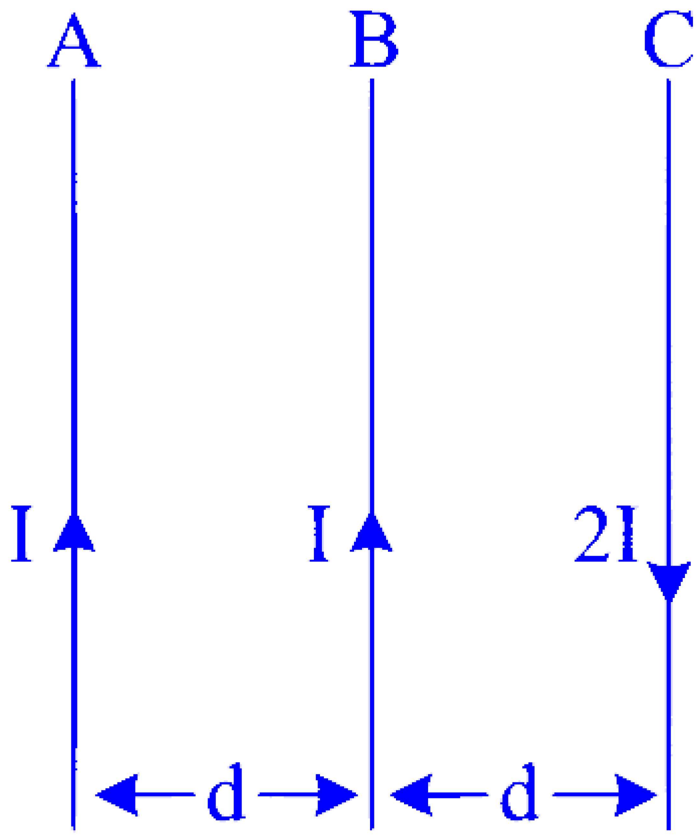
$\mathrm{R} = 3 \times 2 = 6\Omega$

Since, the potentiometer is balanced. Voltage across wire segment $= 1.5\ \mathrm{V}$

$\therefore \quad \mathrm{IR} = 1.5\ \mathrm{V}$
$\therefore \quad \mathrm{I} = \frac{1.5}{6} = 250\ \mathrm{mA}$

Question 109

A, B and C are three parallel conductors of equal lengths and carry currents I, I and 2I respectively as shown in figure. Distance AB and BC is same as 'd'. If 'F_1' is the force exerted by B on A and F_2 is the force exerted by C on A, then



Options:

A. $F_1 = F_2$

B. $F_1 = -F_2$

C. $F_1 = 2F_2$

D. $\mathrm{F}_1 = \frac{1}{2}\mathrm{~F}_2$

Answer: B

Solution:

Force per unit length exerted by B on A, $\mathrm{F}_1 = \frac{\mu_0(\mathrm{I})(\mathrm{I})}{2\pi\mathrm{d}} = \frac{\mu_0\mathrm{I}^2}{2\pi\mathrm{d}}$ (outside the plane of paper)

Force per unit length exerted by C on A,

$\mathrm{F}_2 = \frac{\mu_0(\mathrm{I})(2\mathrm{I})}{2\pi(2\mathrm{~d})} = \frac{\mu_0\mathrm{I}^2}{2\pi\mathrm{d}}$ (Inside the plane of paper)

$\therefore \quad F_1 = -F_2$

Question 110

Two electric dipoles of moment P and $27P$ are placed on a line with their centres 24 cm apart. Their dipole moments are in opposite direction. At which point the electric field will be zero between the dipoles from the centre of dipole of moment P?

Options:

A. 6 cm

B. 8 cm

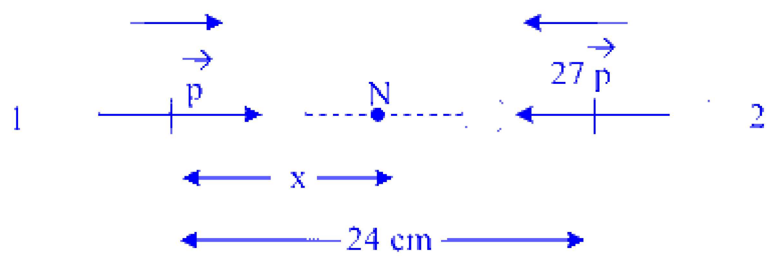
C. 10 cm

D. 12 cm

Answer: A

Solution:

Dipole of moment p is at a distance x from N



At N, |E. F. due to dipole 1| = |E. F. due to dipole 2|

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{x^3}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{2(27p)}{(24-x)^3}$$

$$\therefore \quad \frac{1}{x^3}=\frac{27}{(24-x)^3}\Rightarrow x=6\text{ cm.}$$

Question 111

Converging or diverging ability of a lens or mirror is called

Options:

A. focal power

B. focal length

C. magnifying power

D. linear magnification

Answer: A

Solution:

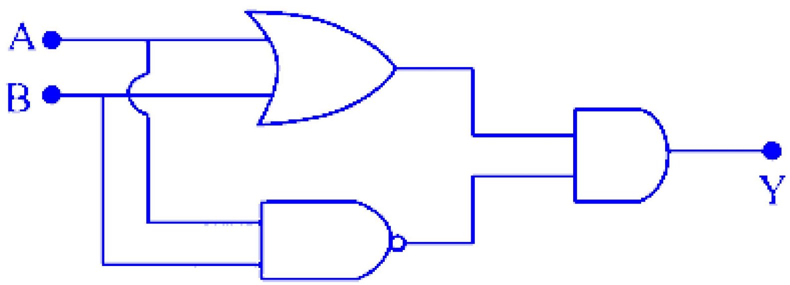
The converging or diverging ability of a lens or mirror is referred to as its "focal power." Focal power, measured in diopters (D), is the reciprocal of the focal length (in meters). It indicates how strongly the lens or mirror converges (positive focal power) or diverges (negative focal power) light.

Therefore, the correct answer is :

Option A : focal power

Question 112

The following logic gate combination is equivalent to



Options:

A. NAND gate

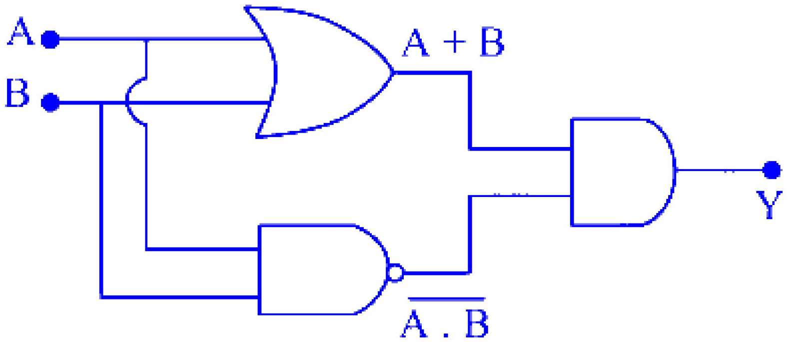
B. OR gate

C. XOR gate

D. NOT gate

Answer: C

Solution:



$\therefore \quad Y = \overline{A \cdot B} \cdot (A + B)$

$\therefore \quad Y = (\overline{A} + \overline{B}) \cdot (A + B) \quad \ldots (\because \overline{A \cdot B} = \overline{A} + \overline{B})$

$\therefore \quad Y = \overline{A} \cdot B + \overline{B} \cdot A$

$\therefore \quad Y = A \cdot \overline{B} + B \cdot \overline{A}$

This represents XOR gate.

Question 113

Radiations of two photons having energies twice and five times the work function of metal are incident successively on metal surface. The ratio of the maximum velocity of photo electrons emitted in the two cases will be

Options:

A. 1 : 1

B. 1 : 2

C. 1 : 3

D. 1 : 4

Answer: B

Solution:

To determine the ratio of the maximum velocities of photoelectrons emitted due to the incident photons, we can apply the photoelectric equation given by Einstein. The kinetic energy (KE) of the emitted photoelectrons can be found by the equation:

$$KE = hf - \phi$$

where hf is the energy of the incident photon, and ϕ is the work function of the metal.

Since we know the energies of the photons are in multiples of the work function (ϕ): the first photon has an energy of 2ϕ and the second has an energy of 5ϕ, we can substitute these into the equation above to find the kinetic energies of the emitted photoelectrons in each case.

For the first photon:

$$KE_1 = 2\phi - \phi = \phi$$

For the second photon:

$$KE_2 = 5\phi - \phi = 4\phi$$

The kinetic energy of a photoelectron is also given by the equation:

$$KE = \frac{1}{2}mv^2$$

where m is the mass of the electron, and v is the velocity of the photoelectron. Therefore, we can equate the expressions for kinetic energy derived from the photoelectric effect to this kinetic energy formula to compare the velocities.

For the first photon:

$$\phi = \frac{1}{2}mv_1^2$$

For the second photon:

$$4\phi = \frac{1}{2}mv_2^2$$

To find the ratio $v_1 : v_2$, we solve for v_1 and v_2 and take the ratio. From $KE = \frac{1}{2}mv^2$, we get $v = \sqrt{\frac{2KE}{m}}$. Therefore, the velocities are:

$$v_1 = \sqrt{\frac{2\phi}{m}}$$

$$v_2 = \sqrt{\frac{2 \cdot 4\phi}{m}} = \sqrt{4}\sqrt{\frac{2\phi}{m}} = 2\sqrt{\frac{2\phi}{m}}$$

So, the ratio of v_1 to v_2 is $\frac{\sqrt{\frac{2\phi}{m}}}{2\sqrt{\frac{2\phi}{m}}} = \frac{1}{2}$.

Hence, the correct option is **Option B: 1 : 2**.

Question 114

Time period of simple pendulum on earth's surface is 'T'. Its time period becomes 'xT' when taken to a height R (equal to earth's radius) above the earth's surface. Then the value of 'x' will be

Options:

A. 4

B. 2

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: B

Solution:

$T = 2\pi\sqrt{\frac{l}{g}}$

At a height 'h' from earth's surface,

$$xT = 2\pi\sqrt{\frac{l}{g_h}}$$

$$\therefore \quad x = \sqrt{\frac{g}{g_h}} \quad ..... \text{(i)}$$

Now, $g_h = \frac{GM}{(R+h)^2}$

$\therefore \quad g_h = \frac{GM}{4R^2} \quad .....(\because h = R)$

$\therefore \quad g_h = \frac{g}{4} \quad .... \text{(ii)}$

$\therefore$ From equations (i) and (ii),

$x = \sqrt{\frac{g}{g/4}} = \sqrt{4} = 2$

Question 115

Consider a soap film on a rectangular frame of wire of area $3 \times 3 \text{ cm}^2$. If the area of the soap film is increased to $5 \times 5 \text{ cm}^2$, the work done in

the process will be (surface tension of soap solution is 2.5×10^{-2} N/m)

Options:

A. 9×10^{-6} J

B. 16×10^{-6} J

C. 40×10^{-6} J

D. 80×10^{-6} J

Answer: D

Solution:

$A_1 = 9 \times 10^{-4}\ \mathrm{m}^2,\ A_2 = 25 \times 10^{-4}\ \mathrm{m}^2$
$T = 2.5 \times 10^{-2}\ \mathrm{N/m}$

Work done,

$W = 2\,T\Delta A = 2 \times 2.5 \times 10^{-2} \times (25 - 9) \times 10^{-4}$
$W = 80 \times 10^{-6}$ J

The rectangular frame has two surfaces. Hence, the formula for work done contains the factor of ' 2 '.

Question 116

In Lyman series, series limit of wavelength is λ_1. The wavelength of first line of Lyman series is λ_2 and in Balmer series, the series limit of wavelength is λ_3. Then the relation between λ_1, λ_2 and λ_3 is

Options:

A. $\lambda_1 = \lambda_2 + \lambda_3$

B. $\lambda_2 = \lambda_1 + \lambda_3$

C. $\frac{1}{\lambda_1} = \frac{1}{\lambda_2} - \frac{1}{\lambda_3}$

D. $\frac{1}{\lambda_1} - \frac{1}{\lambda_2} = \frac{1}{\lambda_3}$

Answer: D

Solution:

According to Rydberg's formula,

$\frac{1}{\lambda} = \mathrm{R}\left(\frac{1}{\mathrm{n}^2} - \frac{1}{\mathrm{m}^2}\right)$

For series limit of Lyman series,

$$\mathrm{n} = 1,\ \mathrm{m} = \infty, \lambda = \lambda_1$$
$$\therefore \quad \frac{1}{\lambda_1} = \mathrm{R}$$

For 1^{st} line of Lyman series,

$$\mathrm{n} = 1,\ \mathrm{m} = 2, \lambda = \lambda_2$$
$$\therefore \quad \frac{1}{\lambda_2} = \frac{3\mathrm{R}}{4}$$

For series limit of Balmer series,

$$\mathrm{n} = 2,\ \mathrm{m} = \infty, \lambda = \lambda_3$$
$$\therefore \quad \frac{1}{\lambda_3} = \frac{\mathrm{R}}{4}$$

Now, $\frac{1}{\lambda_1} - \frac{1}{\lambda_2} = \mathrm{R} - \frac{3\mathrm{R}}{4} = \frac{\mathrm{R}}{4}$

$\therefore \quad \frac{1}{\lambda_1} - \frac{1}{\lambda_2} = \frac{1}{\lambda_3}$

Question 117

The magnetic moment of a current (I) carrying circular coil of radius 'r' and number of turns 'n' depends on

Options:

A. n only

B. I only

C. r only

D. n, I and r

Answer: D

Solution:

$\mathrm{m}=\mathrm{nI}A$
$\mathrm{m}=\mathrm{nI}\left(\pi \mathrm{r}^2\right)$

Question 118

A spherical drop of liquid splits into 1000 identical spherical drops. If 'E_1' is the surface energy of the original drop and 'E_2' is the total surface energy of the resulting drops, then $\frac{E_1}{E_2}=\frac{x}{10}$. Then value of '$x$' is

Options:

A. 9

B. 7

C. 3

D. 1

Answer: D

Solution:

$$
\begin{aligned}
& r= \text{ Radius of small drop} \\
& R= \text{ Radius of bigger drop} \\
\therefore \quad & \frac{4}{3} \pi R^3=\frac{4}{3} \pi(1000) r^3 \\
\therefore \quad & R=10 r \\
& E_1=T_1^2=T\left(4 \pi R^2\right) \\
& E_2=n \mathrm{TA}_2^2=1000 \times T\left(4 \pi r^2\right) \\
\therefore \quad & \frac{E_1}{E_2}=\frac{R^2}{1000 r^2}=\frac{(10 r)^2}{1000 r^2} \\
\therefore \quad & \frac{E_1}{E_2}=\frac{1}{10} \\
\therefore \quad & x=1
\end{aligned}
$$

Question 119

The displacement of two sinusoidal waves is given by the equation

$y_1 = 8\sin(20x - 30t)$
$y_2 = 8\sin(25x - 40t)$

then the phase difference between the waves after time $t = 2$ s and distance $x = 5$ cm will be

Options:

A. 2 radian

B. 3 radian

C. 4 radian

D. 5 radian

Answer: D

Solution:

$y_1 = 8\sin(20x - 3t)$

Substituting $x = 5$ cm and t $= 2$ s,

$y_1 = 8\sin(40)$

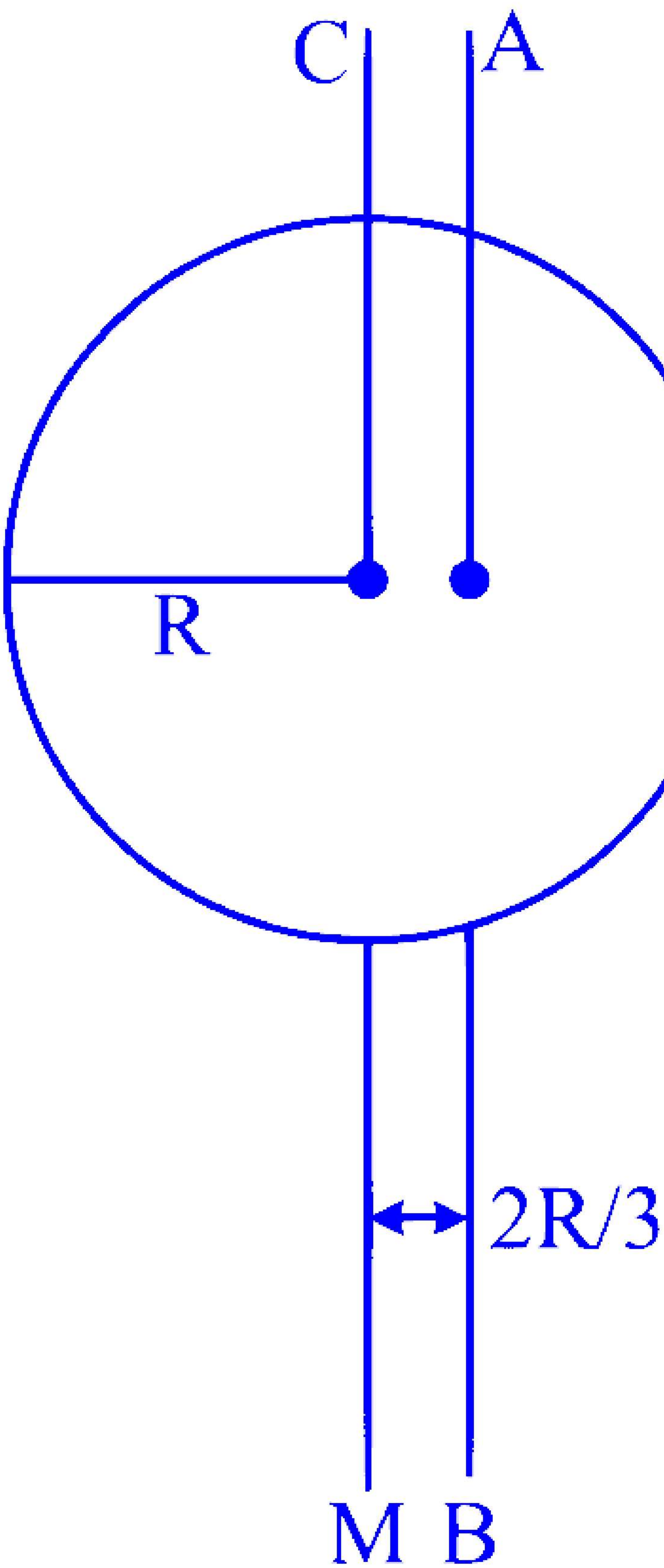
Similarly, $y_2 = 8\sin(45)$

$\therefore$ phase difference $= 45 - 40 = 5$ radian

Question 120

I_1 is the moment of inertia of a circular disc about an axis passing through its centre and perpendicular to the plane of disc. I_2 is its moment of inertia about an axis AB perpendicular to plane and

parallel to axis CM at a distance $\frac{2R}{3}$ from centre. The ratio of I_1 and I_2 is $x : 17$. The value of 'x' is (R = radius of the disc)



Options:

A. 9

B. 12

C. 15

D. 17

Answer: A

Solution:

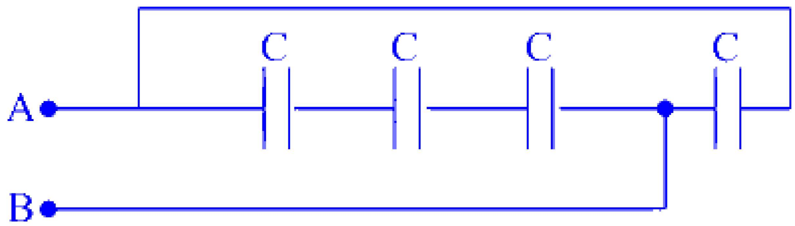
Using Parallel axis theorem, $I_2 = I_1 + Mh^2$

For a disc, $I_1 = \frac{1}{2}MR^2$ and given that, $h = \frac{2R}{3}$

$$\therefore \quad I_2 = \frac{1}{2}MR^2 + M\left(\frac{4R^2}{9}\right)$$
$$\therefore \quad I_2 = \frac{1}{2}MR^2\left(1 + \frac{8}{9}\right) = I_1 \times \frac{17}{9}$$
$$\therefore \quad \frac{I_1}{I_2} = \frac{9}{17}$$

Question 121

The equivalent capacity between terminal A and B is



Options:

A. $\frac{C}{4}$

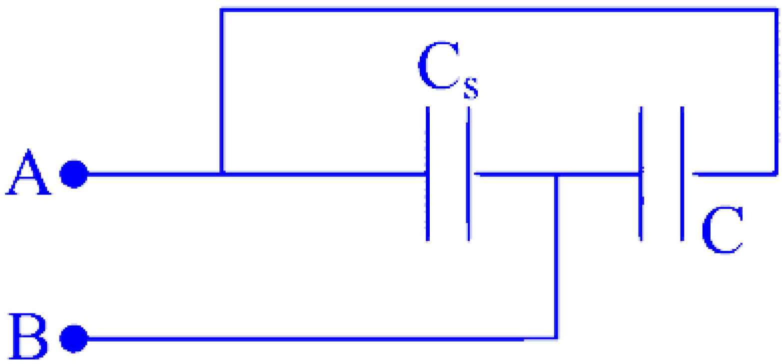
B. $\frac{3C}{4}$

C. $\frac{C}{3}$

D. $\frac{4C}{3}$

Answer: D

Solution:



$$\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$

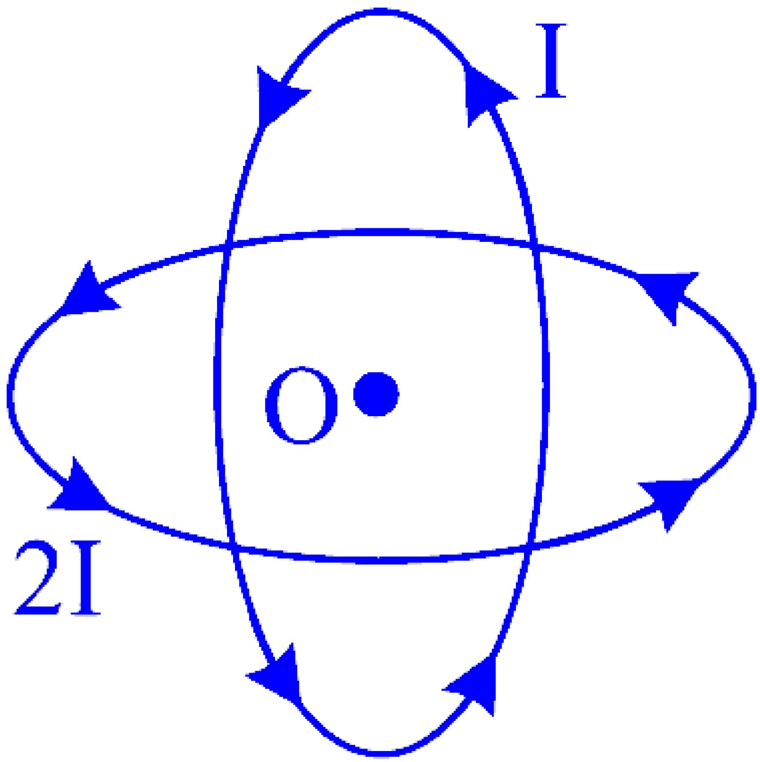
$$\therefore \quad C_s = \frac{C}{3}$$

Now, C_s and C are connected in parallel,

$$\therefore \quad C_{net} = C_s + C = \frac{C}{3} + C = \frac{4C}{3}$$

Question 122

Two similar coils each of radius R are lying concentrically with their planes at right angles to each other. The current flowing in them are I and 2I. The resultant magnetic field of induction at the centre will be (μ_0 = Permeability of vacuum)



Options:

A. $\frac{\mu_0 \mathrm{I}}{2 \mathrm{R}}$

B. $\frac{\mu_0 \mathrm{I}}{\mathrm{R}}$

C. $\frac{3 \mu_0 \mathrm{I}}{2 \mathrm{R}}$

D. $\frac{\sqrt{5} \mu_0 \mathrm{I}}{2 \mathrm{R}}$

Answer: D

Solution:

$B_1=\frac{\mu_0 I}{2 R}, B_2=\frac{\mu_0(2 I)}{2 R}$

Resultant magnetic field, $B=\sqrt{B_1+B_2}$

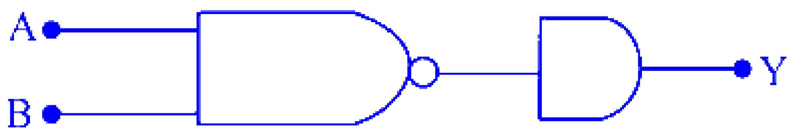
$\therefore \quad B=\sqrt{\left(\frac{\mu_0 I}{2 R}\right)^2+\left(\frac{\mu_0(2 I)}{2 R}\right)^2}$

$\therefore \quad B=\frac{\mu_0 I}{2 R} \sqrt{1^2+2^2}$

$\therefore \quad B=\frac{\sqrt{5} \mu_0 I}{2 R}$

Question 123

The logic gate combination circuit shown in the figure performs the logic function of



Options:

A. AND gate

B. NAND gate

C. OR gate

D. XOR gate

Answer: A

Solution:

[Note: The gate at the output in the circuit is AND gate. It requires minimum two inputs. As, there is only one input the gate is not operational.]

Question 124

Two sounding sources send waves at certain temperature in air of wavelength $50 \mathrm{~cm}$ and $50.5 \mathrm{~cm}$ respectively. The frequency of sources differ by $6 \mathrm{~Hz}$. The velocity of sound in air at same temperature is

Options:

A. $300 \mathrm{~m/s}$

B. $303 \mathrm{~m/s}$

C. $313 \mathrm{~m/s}$

D. 330 m/s

Answer: B

Solution:

$v = n\lambda$

Since, both the sound sources are at same temperature, velocity of sound in both cases would be the same.

$\therefore \quad v = (50n_1)\text{cm/s} \quad \text{.... (i)}$

$v = (50.5n_2)\text{cm/s} \quad \text{.... (ii)}$

$\frac{n_1}{n_2} = \frac{50.5}{50} \quad \text{...[From (i) and (ii)]}$

$\therefore \quad \frac{n_1 - n_2}{n_2} = \frac{50.5-50}{50}$

$\therefore \quad \frac{6}{n_2} = \frac{0.50}{50} = \frac{1}{100} \quad \ldots.(\because n_1 - n_2 = 6 \text{ Hz})$

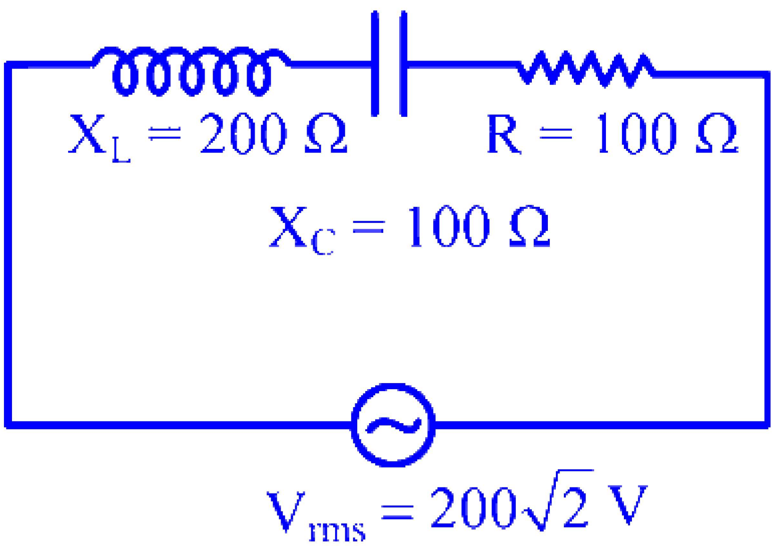
$\therefore \quad n_2 = 600 \text{ Hz}$

$\therefore \quad v = \frac{50.5 \times 600}{100} \text{ m/s} \quad \ldots.[\text{ From (ii) }]$

$\therefore \quad v = 303 \text{ m/s}$

Question 125

In the given circuit, r.m.s. value of current through the resistor R is



Options:

A. 2 A

B. 0.5 A

C. 20 A

D. $2\sqrt{2}$ A

Answer: A

Solution:

$$Z=\sqrt{R^2+(X_L-X_C)^2}$$

$$Z=\sqrt{100^2+100^2}$$

$$Z=100\sqrt{2}\Omega$$

$$i_{rms}=\frac{V_{rms}}{Z}$$

$$\therefore \quad i_{rms}=\frac{200\sqrt{2}}{100\sqrt{2}}$$

$$i_{rms}=2\text{ A}$$

Question 126

A particle of mass 'm' moving east ward with a speed 'v' collides with another particle of same mass moving north-ward with same speed 'v'. The two particles coalesce after collision. The new particle of mass '2 m' will move in north east direction with a speed (in m/s)

Options:

A. V

B. 2 V

C. $\frac{V}{2}$

D. $\frac{V}{\sqrt{2}}$

Answer: D

Solution:

Momentum of particle moving towards east

$\overrightarrow{p_1} = mv\hat{i}$

Momentum of particle moving towards North

$\overrightarrow{p_2} = mv\hat{j}$

Momentum after collision,

$\vec{p} = 2m\left(v_x\hat{i} + v_y\hat{j}\right)$

Applying momentum conservation,

$$\begin{aligned} &\overrightarrow{p_1} + \overrightarrow{p_2} = \vec{p} \\ &mv\hat{i} + mv\hat{j} = 2m\left(v_x\hat{i} + v_y\hat{j}\right) \\ \therefore\quad &2mv_x = mv \\ &v_x = \frac{v}{2} \end{aligned}$$

Similarly, $v_y = \frac{v}{2}$

$v_R = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(\frac{v}{2}\right)^2 + \left(\frac{v}{2}\right)^2} = \frac{v}{\sqrt{2}}$

Question 127

The height at which the weight of the body becomes $\left(\frac{1}{9}\right)^{\text{th}}$ its weight on the surface of earth is $(R =$ radius of earth)

Options:

A. 8R

B. 4R

C. 3R

D. 2R

Answer: D

Solution:

$$
\begin{aligned}
& W_h = \frac{W}{9} \\
& mg_h = \frac{mg}{9} \\
& g_h = \frac{g}{9} \\
\therefore \quad & \frac{GM}{(R+h)^2} = \frac{GM}{9R^2} \\
\therefore \quad & R + h = 3R \\
\therefore \quad & h = 2R
\end{aligned}
$$

Question 128

A single turn current loop in the shape of a right angle triangle with side 5 cm, 12 cm, 13 cm is carrying a current of 2 A. The loop is in a uniform magnetic field of magnitude 0.75 T whose direction is parallel to the current in the 13 cm side of the loop. The magnitude of the magnetic force on the 5 cm side will be $\frac{x}{130}$ N. The value of 'x' is

Options:

A. 4

B. 9

C. 12

D. 15

Answer: B

Solution:

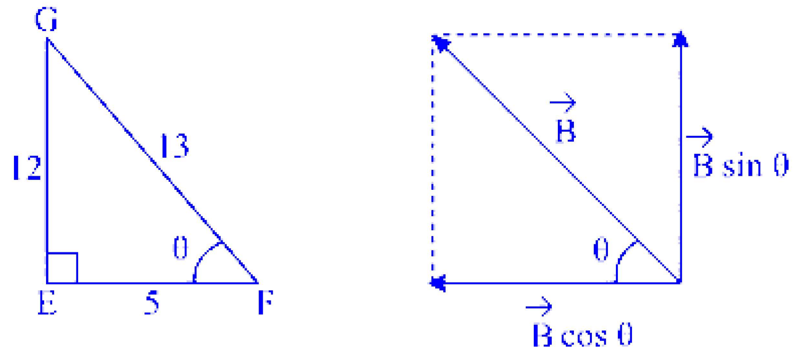


Figure (a) Figure (b)

The net magnetic field is acting in the direction of GF as shown in figures.

Resolving $\overrightarrow{B}$ into its components, amongst the components, only $\vec{B}\sin\theta$ exerts force on side EF of current carrying loop.

$\therefore \quad F_{EF} = I \times d(EF) \times B\sin\theta$

From figure (a), $\sin\theta = \frac{12}{13}$

$\therefore \quad F_{EF} = 2 \times 0.05 \times 0.75 \times \frac{12}{13}$

$\therefore \quad F_{EF} = \frac{9}{130}\ N$

$\therefore \quad x = 9$

Question 129

41 tuning forks are arranged in increasing order of frequency such that each produces 5 beats/second with next tuning fork. If frequency of last tuning fork is double that of frequency of first fork. Then frequency of first and last fork is

Options:

A. 400, 200 Hz

B. 200, 400 Hz

C. 100, 200 Hz

D. 205, 410 Hz

Answer: B

Solution:

Let Frequency of 1^{st} tuning fork be $= n_1$

$\therefore$ frequency of 41^{st} tuning fork $= n_{41}$

Now,

$$\begin{aligned} & n_{41} = n_1 + (41-1) \times 5 \\ & \text{But, } n_{41} = 2n_1 \\ \therefore \quad & 2n_1 = n_1 + 200 \\ \therefore \quad & n_1 = 200 \text{ Hz} \\ \therefore \quad & n_{41} = 400 \text{ Hz} \end{aligned}$$

Question 130

In two separate setups for Biprism experiment using same wavelength, fringes of equal width are obtained. If ratio of slit separation is 2 : 3 then the ratio of the distance between the slit and screen in the two setups is

Options:

A. 2 : 3

B. 1 : 2

C. 4 : 9

D. 9 : 4

Answer: A

Solution:

$$\begin{aligned} & \text{Fringe width, } W = \frac{\lambda D}{d} \\ & \text{For constant } W \text{ and } \lambda \\ & D \propto d \\ \therefore \quad & \frac{D_1}{D_2} = \frac{d_1}{d_2} = \frac{2}{3} \end{aligned}$$

Question 131

A composite slab consists of two materials having coefficient of thermal conductivity K and $2\ \mathrm{K}$, thickness x and $4\mathrm{x}$ respectively. The temperature of the two outer surfaces of a composite slab are T_2 and $\mathrm{T}_1\ (\mathrm{T}_2 > \mathrm{T}_1)$. The rate of heat transfer through the slab in a steady state is $\left[\frac{A(T_2-T_1)K}{x}\right] \cdot f$ where 'f' is equal to

Options:

A. 1

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $\frac{1}{3}$

Answer: D

Solution:

$$R_{eq} = R_1 + R_2$$
$$\because \quad R_1 = \frac{x}{KA}, R_2 = \frac{4x}{2KA}$$
$$\therefore \quad R_{eq} = \frac{x}{KA} + \frac{2x}{KA} = \frac{3x}{KA}$$

Rate of heat transfer of composite slab is given by,

$$\frac{dQ}{dt} = \frac{T_2-T_1}{R_{eq}} = \frac{KA(T_2-T_1)}{3x}$$

$$\therefore \quad f = \frac{1}{3}$$

Question 132

A black sphere has radius 'R' whose rate of radiation is 'E' at temperature 'T'. If radius is made $R/3$ and temperature '$3T$', the rate

of radiation will be

Options:

A. E

B. 3E

C. 6E

D. 9E

Answer: D

Solution:

$E = eA\sigma T^4$

But for sphere, $A = 4\pi R^2$

$$\therefore \quad E = e\left(4\pi R^2\right)\sigma T^4$$
$$\therefore \quad \frac{E_1}{E_2} = \frac{R_1^2 T_1^4}{R_2^2 T_2^4}$$
$$\therefore \quad \frac{E}{E_2} = \frac{R^2 T^4}{\left(\frac{R}{3}\right)^2 (3T)^4}$$
$$\therefore \quad E_2 = 9E$$

Question 133

The potential on the plates of capacitor are $+20$ V and -20 V. The charge on the plate is 40C. The capacitance of the capacitor is

Options:

A. 2 F

B. 1 F

C. 4 F

D. 0.5 F

Answer: B

Solution:

$$V = 20 - (-20) = 40 \text{ volt}$$
$$Q = 40C$$
$$C = \frac{Q}{V}$$
$$\therefore \quad C = \frac{40}{40}$$
$$\therefore \quad C = 1F$$

Question 134

A thin uniform circular disc of mass 'M' and radius 'R' is rotating with angular velocity 'ω', in a horizontal plane about an axis passing through its centre and perpendicular to its plane. Another disc of same radius but of mass $\left(\frac{M}{2}\right)$ is placed gently on the first disc co-axially. The new angular velocity will be

Options:

A. $\frac{2}{3}\omega$

B. $\frac{4}{5}\omega$

C. $\frac{5}{4}\omega$

D. $\frac{3}{2}\omega$

Answer: A

Solution:

Angular momentum $= \mathrm{I}\omega$

By conservation of angular momentum, $\mathrm{I}_1\omega_1 = \mathrm{I}_2\omega_2$

Here, $I_1 = \frac{MR^2}{2}, I_2 = \frac{(M+M/2)}{2}R^2 = \frac{3MR^2}{4}$

$$\therefore \quad \frac{MR^2}{2}\omega_1 = \frac{3MR^2}{4}\omega_2$$
$$\therefore \quad \omega_2 = \frac{2}{3}\omega_1$$

Question 135

A gas at normal temperature is suddenly compressed to one-fourth of its original volume. If $\frac{C_p}{C_v} = \gamma = 1.5$, then the increase in its temperature is

Options:

A. 273 K

B. 373 K

C. 473 K

D. 573 K

Answer: A

Solution:

Given that, $V_2 = \frac{V_1}{4}, \frac{C_p}{C_v} = \gamma = 1.5$

As the process is sudden, it is an adiabatic expansion,

$$\therefore \quad T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$
$$\therefore \quad T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$
$$= T_1(4)^{\gamma-1}$$
$$= T_1 \times (4)^{0.5}$$
$$= 2\,T_1$$
$$T_2 - T_1 = T_1$$
$$T_2 - T_1 = 273\text{ K} \quad (\because T_1 = \text{ Normal temperature })$$

Question 136

When light of wavelength λ is incident on a photosensitive surface the stopping potential is 'V'. When light of wavelength 3λ is incident on same surface the stopping potential is $\frac{V}{6}$. Then the threshold wavelength for the surface is

Options:

A. 2λ

B. 3λ

C. 4λ

D. 5λ

Answer: D

Solution:

$$\lambda_1 = \lambda, (V_0)_1 = V$$
$$\lambda_2 = 3\lambda, (V_0)_2 = \frac{V}{6}$$

Photo electric equation is given by

$eV_0 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$ ..... (i)

In first case,

$eV = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$

For second case,

$\frac{eV}{6} = hc\left(\frac{1}{3\lambda} - \frac{1}{\lambda_0}\right)$ ..... (ii)

Dividing equation (i) by equation (ii),

$$6 = \frac{\frac{1}{\lambda} - \frac{1}{\lambda_0}}{\frac{1}{3\lambda} - \frac{1}{\lambda_0}} = \frac{3\lambda_0 - \lambda}{\lambda_0 - 3\lambda}$$
$$\therefore \quad \lambda_0 = 5\lambda$$

Question 137

One of the necessary condition for total internal reflection to take place is

(i = angle of incidence, $\mathrm{i_c}$ = critical angle)

Options:

A. $\mathrm{i} < \mathrm{i_c}$

B. $\mathrm{i} = \mathrm{i_c}$

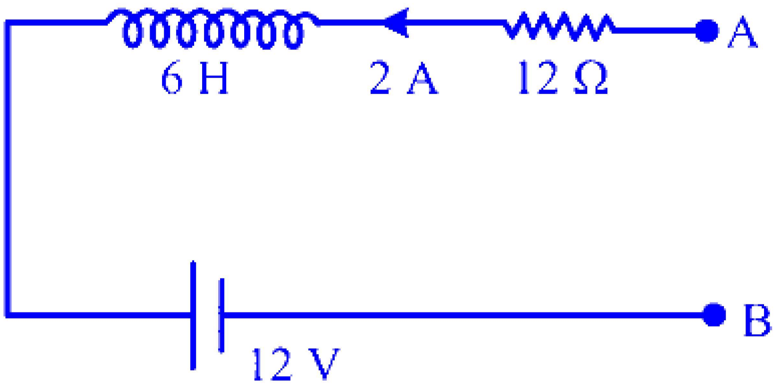
C. $i = \frac{\pi^{e}}{2}$

D. $i > i_c$

Answer: D

Question 138

In the given circuit, if $\frac{\mathrm{dI}}{\mathrm{dt}} = -1\ \mathrm{A/s}$ then the value of $(V_A - V_B)$ at this instance will be



Options:

A. 30 V

B. 24 V

C. 18 V

D. 9 V

Answer: A

Solution:

Applying KVL in the given circuit from point A to B,

$$V_{AB} - IR - L\frac{dI}{dt} - 12 = 0$$
$$V_{AB} - (2)(12) - 6(-1) - 12 = 0$$
$$V_{AB} = 24 + 12 - 6$$
$$\therefore \quad V_{AB} = 30\ V$$

Question 139

An inductor of $0.5\ mH$, a capacitor of $20\ \mu F$ and a resistance of 20Ω are connected in series with a $220\ V$ a.c. source. If the current is in phase with the e.m.f. the maximum current in the circuit is $\sqrt{x}A$. The value of 'x' is

Options:

A. 44

B. 82

C. 146

D. 242

Answer: D

Solution:

When current is in phase with voltage, we have

$$Z = R = 20\Omega$$
$$e_0 = \sqrt{2}e_{rms} = 220\sqrt{2}\ V$$
$$i_0 = \frac{e_0}{Z} = \frac{220\sqrt{2}}{20} = 11\sqrt{2}\ A$$
$$i_0 = \sqrt{242}\ A$$

Question 140

The wavelength of radiation emitted is 'λ_0' when an electron jumps from the second excited state to the first excited state of hydrogen atom. If the electron jumps from the third excited state to the second orbit of the hydrogen atom, the wavelength of the radiation emitted will be $\frac{20}{x}\lambda_0$. The value of x is

Options:

A. 3

B. 9

C. 13

D. 27

Answer: D

Solution:

According to Rydberg's formula,

$\frac{1}{\lambda} = \mathrm{R}\left(\frac{1}{\mathrm{n}_1^2} - \frac{1}{\mathrm{n}_2^2}\right)$ ...... (i)

When electron jumps from 2^{nd} exited state to first exited state,

$\mathrm{n}_2 = 3, \mathrm{n}_1 = 2, \lambda = \lambda_0$, we get

$\frac{1}{\lambda_0} = \mathrm{R}\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

When electron jumps from 3^{rd} exited state to 2^{nd} orbit,

$\mathrm{n}_2 = 4, \mathrm{n}_1 = 2$, we get

$\frac{1}{\lambda} = \mathrm{R}\left(\frac{1}{4^2} - \frac{1}{2^2}\right)$

$$\therefore \quad \frac{\lambda}{\lambda_0}=\frac{R\left(\frac{1}{2^2}-\frac{1}{3^2}\right)}{R\left(\frac{1}{2^2}-\frac{1}{4^2}\right)}$$
$$=\frac{5}{36}\times\frac{16}{3}=\frac{20}{27}$$
$$\therefore \quad \lambda=\frac{20}{27}\lambda_0$$
$$\Rightarrow x=27$$

Question 141

Two particles having mass 'M' and 'm' are moving in a circular path with radius 'R' and 'r' respectively. The time period for both the particles is same. The ratio of angular velocity of the first particle to the second particle will be

Options:

A. 1 : 1

B. 1 : 2

C. 2 : 3

D. 3 : 4

Answer: A

Solution:

$$T=\frac{2\pi}{\omega}$$
Hence, $T_1=T_2$
$$\Rightarrow \omega_1=\omega_2$$
$$\therefore \quad \frac{\omega_1}{\omega_2}=\frac{1}{1}$$

Question 142

The excess pressure inside a first spherical drop of water is three times that of second spherical drop of water. Then the ratio of mass of first spherical drop to that of second spherical drop is

Options:

A. 1 : 3

B. 1 : 6

C. 1 : 9

D. 1 : 27

Answer: D

Solution:

Excess pressure inside the 1^{st} spherical drop is given by,

$P_1 = \frac{2T}{r_1}$

Similarly, for 2^{nd} drop

$$P_2 = \frac{2T}{r_2}$$

$$P_1 = 3P_2 \quad \text{.....(Given)}$$

$$\therefore \frac{2T}{r_1} = 3\left(\frac{2T}{r_2}\right)$$

$$\therefore \frac{r_1}{r_2} = \frac{1}{3}$$

Now, $\frac{m_1}{m_2} = \frac{V_1\rho_1}{V_2\rho_2}$

$$\therefore \frac{m_1}{m_2} = \frac{V_1}{V_2}$$

As both the drops are of water, $\rho_1 = \rho_2$ $\therefore \frac{m_1}{m_2} = \frac{4/3\pi r_1^3}{4/3\pi r_2^3}$

$$\therefore \frac{m_1}{m_2} = \frac{1}{27}$$

Question 143

When forward bias is applied to a p-n junction, then what happens to the potential barrier (V_B) and the width (X) of the depletion region?

Options:

A. V_B increase, X decreases

B. V_B decreases, X increase

C. V_B increase, X increase

D. V_B decreases, X decreases

Answer: D

Question 144

Two inductors of $60 \mathrm{~mH}$ each are joined in parallel. The current passing through this combination is $2.2 \mathrm{~A}$. The energy stored in this combination of inductors in joule is

Options:

A. 0.0333

B. 0.0667

C. 0.0726

D. 0.0984

Answer: C

Solution:

$L_1 = L_2 = L = 60 \mathrm{~mH}$

When two inductors are connected in parallel, their equivalent inductance is given by,

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$
$$\therefore \quad L_{eq} = \frac{L}{2} = 30 \text{ mH}$$
$$u_B = \frac{1}{2} L_{eq} I^2$$
$$\therefore \quad u_B = \frac{1}{2} \times 30 \times 10^{-3} \times 2.2 \times 2.2$$
$$\therefore \quad u_B = 0.0726 \text{ J}$$

Question 145

A beam of light is incident on a glass plate at an angle of 60°. The reflected ray is polarized. If angle of incidence is 45° then angle of refraction is

Options:

A. $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

B. $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

C. $\sin^{-1}\left(\sqrt{\frac{3}{2}}\right)$

D. $\cos^{-1}\left(\sqrt{\frac{3}{2}}\right)$

Answer: A

Solution:

According to Brewster's law,

$$\tan \theta_B = n$$
$$\therefore \quad \tan 60^\circ = n$$

$\therefore \quad n = \sqrt{3}$

Now, $\frac{\sin i}{\sin r} = n$

$\therefore \quad \sin r = \frac{\sin i}{n}$

$\therefore \quad \sin r = \frac{\sin 45^\circ}{\sqrt{3}}$

$\therefore \quad \sin r = \frac{1}{\sqrt{6}} \quad \ldots \left(\because \sin 45^\circ = \frac{1}{\sqrt{2}}\right)$

$\therefore \quad r = \sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

Question 146

Consider a light planet revolving around a massive star in a circular orbit of radius 'r' with time period 'T'. If the gravitational force of attraction between the planet and the star is proportional to $r^{\frac{7}{2}}$, then T^2 is proportional to

Options:

A. $r^{9/2}$

B. $r^{7/2}$

C. $r^{5/2}$

D. $r^{3/2}$

Answer: A

Solution:

For the planet to orbit around the star, the centripetal force must be provided by gravitational force. Hence, $F_G = F_a$.

$F_a \propto -r^{-7/2}$ .... (Given)

(−ve sign indicates force is towards the centre of orbit)

Hence, $a \propto -r^{-7/2}$

$\therefore \quad -\omega^2 r \propto -r^{-7/2}$

$\therefore \quad \omega^2 \propto r^{-9/2}$

$\therefore \quad \frac{4\pi^2}{T^2} \propto r^{-9/2}$

$\Rightarrow T^2 \propto r^{9/2}$

Question 147

A potentiometer wire has length of $5 \mathrm{~m}$ and resistance of 16Ω. The driving cell has an e.m.f. of $5 \mathrm{~V}$ and an internal resistance of 4Ω. When the two cells of e.m.f.s $1.3 \mathrm{~V}$ and $1.1 \mathrm{~V}$ are connected so as to assist each other and then oppose each other, the balancing lengths are respectively

Options:

A. $3 \mathrm{~m}, 0.25 \mathrm{~m}$

B. $0.25 \mathrm{~m}, 3 \mathrm{~m}$

C. $2.5 \mathrm{~m}, 0.3 \mathrm{~m}$

D. $0.3 \mathrm{~m}, 2.5 \mathrm{~m}$

Answer: A

Solution:

$$K=\frac{E R}{(R+r) L}$$

$\mathrm{E}=5 \mathrm{~V}, \mathrm{r}=4 \Omega, \mathrm{L}=5 \mathrm{~m}, \mathrm{R}=16 \Omega$

$$\therefore \quad \mathrm{K}=\frac{5 \times 16}{(16+4) \times 5}$$

$$\therefore \quad \mathrm{K}=0.8 \mathrm{~V} / \mathrm{m}$$

When 'E_1' and 'E_2' are connected so as to assist each other

$$\mathrm{E}_1+\mathrm{E}_2=\mathrm{K}_1$$

$$1.3+1.1=0.8 \times l_1$$

$$\therefore \quad l_1=3 \mathrm{~m}$$

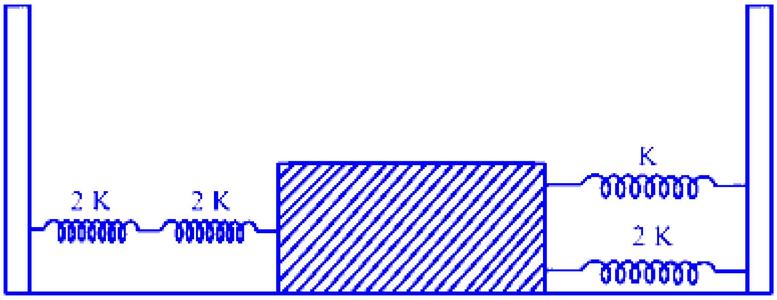
When 'E_1' and 'E_2' are connected so as to oppose each other,

$$E_1 - E_2 = Kl_2$$
$$1.3 - 1.1 = 0.8 \times l_2$$
$$\therefore \quad l_2 = 0.25 \text{ m}$$

As, value for balancing lengths are different in all the options. It is sufficient to calculate balancing length in any one case (Assisting/ opposing) to reach the final correct answer.

Question 148

Four massless springs whose force constants are $2\text{ K}, 2\text{ K}, \text{ K}$ and 2 K respectively are attached to a mass M kept on a frictionless plane as shown in figure, If mass M is displaced in horizontal direction then frequency of oscillating system is



Options:

A. $\frac{1}{2\pi}\sqrt{\frac{\text{K}}{4\text{M}}}$

B. $\frac{1}{2\pi}\sqrt{\frac{4\text{ K}}{\text{M}}}$

C. $\frac{1}{2\pi}\sqrt{\frac{\text{K}}{7\text{M}}}$

D. $\frac{1}{2\pi}\sqrt{\frac{7\text{ K}}{\text{M}}}$

Answer: B

Solution:

On the right hand side of the block, springs are connected in parallel

$\therefore$ Their effective spring constant is given by

$$K_1 = K + 2K$$
$$K_1 = 3K$$

On the left hand side of the block, springs are connected in series.

$\therefore$ Their effective spring constant is given by,

$$\frac{1}{K_2} = \frac{1}{2K} + \frac{1}{2K}$$
$$\therefore \quad K_2 = K$$

$\therefore$ Effective spring constant of the system is given by,

$$K_E = 3K + K = 4K$$
$$\therefore \quad \omega = \sqrt{\frac{K_E}{M}} = \sqrt{\frac{4K}{M}}$$
$$\therefore \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{4K}{M}}$$

Question 149

About black body radiation, which of the following is the wrong statement?

Options:

A. For all wavelengths, intensity is same.

B. For shorter wavelengths, intensity is more.

C. For longer wavelengths, intensity is less.

D. All wavelengths are emitted by a black body.

Answer: A

Question 150

Two coils have a mutual inductance of 0.004 H. The current changes in the first coil according to equation $I = I_0 \sin \omega t$, where $I_0 = 10$ A and

$\omega = 50\ \pi$ rad s^{-1}. The maximum value of e.m.f. in the second coil in volt is

Options:

A. 5π

B. 4π

C. 2.5π

D. 2π

Answer: D

Solution:

$$|e_s| = M\frac{dI_p}{dt}$$
$$|e_s| = M\frac{d}{dt}I_0 \sin \omega t$$
$$|e_s| = MI_0\omega \cos \omega t$$
$$\therefore \quad |e_s|_{\max} = MI_0\omega = 0.004 \times 10 \times 50\pi$$
$$\therefore \quad |e_s|_{\max} = (2\pi) \text{ volt}$$